AP Practice Exam IV

A 1. $\lim_{x \to b} \frac{b-x}{\sqrt{x}-\sqrt{b}} = \lim_{x \to b} \frac{-1}{\frac{1}{2\sqrt{x}}} = \lim_{x \to b} -2\sqrt{x} = -2\sqrt{b}$

"$\frac{0}{0}$"

A 2. $y' = -\sin x + \sec^2(2x)2$

$y'|_{x=0} = -\sin 0 + 2\sec^2(0) = 0 + 2(1)$
$= 2$

$y - 1 = 2(x-0) \rightarrow y = 2x+1$

C 3. $g'(x) = f(x)$

$g''(x) = f'(x)$ changes sign at $x=2$ + $x=6$

D 4. corner at $x=2 \rightarrow f'(2)$ undefined

D 5. $u = 3x+5$
$\frac{du}{dx} = 3$
$dx = \frac{du}{3}$

$\int u^2 \frac{du}{3} = \frac{1}{3}\left[\frac{1}{3}u^3\right] + c$

$= \frac{1}{9}(3x+5)^3 + C$

B 6. avg $v = \frac{\ln(e) - \ln(1)}{e-1} = \frac{1-0}{e-1} = \frac{1}{e-1}$

A 7. $e^x + e^y \frac{dy}{dx} = 2 + 2\frac{dy}{dx}$

$\frac{dy}{dx}(e^y - 2) = 2 - e^x$

$\frac{dy}{dx} = \frac{2-e^x}{e^y-2}$

$\frac{dy}{dx}\Big|_{(0,0)} = \frac{2-e^0}{e^0-2} = \frac{2-1}{1-2} = -1$

A 8. $f'(x) = 3\ln x + 3x\left(\frac{1}{x}\right)$

$= 3\ln x + 3$

$= \ln x^3 + 3$

C 9. $\int_{-3}^{-2} -x-2\,dx + \int_{-2}^{3} x+2\,dx =$

$-\frac{1}{2}x^2 - 2x\Big]_{-3}^{-2} + \left[\frac{1}{2}x^2 + 2x\right]_{-2}^{3} =$

$-\frac{1}{2}(4) + 4 + \frac{1}{2}(9) - 6 + \frac{9}{2} + 6 - \frac{1}{2}(4) + 4 =$

$-2 + 4 + \frac{9}{2} - 6 + \frac{9}{2} + 6 - 2 + 4 =$
$4 + \frac{18}{2} = 4 + 9 = 13$

B 10. $x^2 - 2kx = 0$
$x(x-2k) = 0$
$x = 0 \quad x = 2k$

$\int_0^{2k} 0 - x^2 + 2kx\,dx = 36$

$-\frac{1}{3}x^3 + kx^2\Big]_0^{2k} = 36$

$-\frac{8k^3}{3} + 4k^3 = 36$
$-8k^3 + 12k^3 = 72$
$4k^3 = 108$
$k^3 = 27$

$k = 3$

D 11. $f'(x) = e^x - 2 = 0$
$e^x = 2$
$\ln 2 = x$
$f(\ln 2) = e^{\ln 2} - 2\ln 2$

$= 2 - \ln 4$

$f''(x) = e^x$

$\left.\begin{array}{l} f'(\ln 2) = 0 \\ f''(\ln 2) > 0 \end{array}\right\}$ $x = \ln 2$ yields a minimum

C 12. MVT $\rightarrow f'(c) = \frac{f(7) - f(-3)}{7-(-3)}$

$f'(c) = \frac{2-4}{10}$

$f'(c) = \frac{-2}{10}$

$f'(c) = -\frac{1}{5}$

A 13. $\int_2^4 6t + 2\,dt = 3t^2 + 2t\Big]_2^4 =$

$3(4)^2 + 2(4) - 3(2)^2 - 2(2) =$

$48 + 8 - 12 - 4 =$
$56 - 16 =$
40

B 14. $g'(x) = -(f(x))^{-2} f'(x)$

$= \frac{-f'(x)}{(f(x))^2}$

$g'(2) = \frac{-f'(2)}{(f(2))^2} = \frac{4}{64} = \frac{1}{16}$

D 15. $\int_1^{15} f(t)\,dt = g(15) - g(1) =$

$2(2) + 3(3) + 4(4) + 5(2) =$

$4 + 9 + 16 + 10 = 39$

A 16. $\frac{dV}{dt} = 12$ ft³/sec Find $\frac{dA}{dt}$ when $\underbrace{V = 36\pi \text{ ft}^3 = \frac{4}{3}\pi r^3}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ → $12 = 4\pi(3)^2 \frac{dr}{dt}$

$\frac{1}{3\pi} = \frac{dr}{dt}$

$27 = r^3$
$r = 3$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$= 8\pi(3)\left(\frac{1}{3\pi}\right)$

$= 8$ ft²/sec

D 17. $\int_{-2}^{2} 15-g(x)-g(x)dx=$

$\int_{-2}^{2} 15-2g(x)dx=$

$15(4)-2\int_{-2}^{2} g(x)dx=$

$60-2\int_{-2}^{2} g(x)dx$

C 18. $h'(x)=f'(g(x))g'(x)$

$h'(2)=f'(g(2))g'(2)$

$=f'(1)g'(2)$

$=5\ (3)=15$

$f'(x)=2x+3$

$f'(1)=5$

C 19. $\frac{d}{dx}(c)$ = dollars/item T

$\frac{d}{dx}(c)$ ← change cost T

$\frac{d}{dx}(c)$ F

cost not amount

A 20. f is differentiable → $f'(c)=0$

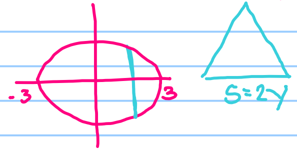
$f''(c)$ may be undefined (ex. $y=x^{4/3}$)

$f''(c)$ may be 0 (ex. $y=x^4$)

D 21. $x^2+y^2=9$

$A=\frac{\sqrt{3}}{4}s^2=\frac{\sqrt{3}}{4}(4y^2)$

$=4(9-x^2)$



$\int_{-3}^{3} \sqrt{3}(9-x^2)dx=$

$\sqrt{3}\left[9x-\frac{1}{3}x^3\right]_{-3}^{3}=$

$\sqrt{3}(27-9+27-9)=$

$36\sqrt{3}$

B 22. $\int_{2}^{x} f(x)dx=y(x)-y(2)$

$\int_{2}^{x} f(x)dx=y(x)-4$

$4+\int_{2}^{x} f(x)dx=y(x)$

A 23. $y'=\frac{1}{\sqrt{1-\left(\frac{3x}{4}\right)^2}}\cdot\frac{3}{4}$

$=\frac{1}{\sqrt{\frac{16-9x^2}{16}}}\cdot\frac{3}{4}$

$=\frac{1}{\frac{\sqrt{16-9x^2}}{4}}\cdot\frac{3}{4}$

$=\frac{3}{\sqrt{16-9x^2}}$

B 24. $\lim_{x\to\infty}\frac{ae^x+b}{e^x+1}=3$

"$\frac{\infty}{\infty}$" $\lim_{x\to\infty}\frac{ae^x}{e^x}=3$

$a=3$

$\lim_{x\to-\infty}\frac{ae^x+b}{e^x+1}=-5$

0 0

$b=-5$

$a+b=3-5=-2$

A 25. $\frac{dy}{dx}=\frac{2y}{x}$

$\int\frac{1}{y}dy=\int\frac{2}{x}dx$

$\ln|y|=2\ln|x|+c$

$\ln|y|=\ln x^2+c$

$y=\pm e^{\ln x^2+c}$

$y=\pm Ce^{\ln x^2}$

$y=Cx^2$

C 26. $f'(x)=\lim_{h\to 0}\frac{\tan(2x+2h)-\tan(2x)}{h}$

$f(x)=\tan(2x)$

$f'(x)=2\sec^2(2x)$

B 27. $\sin\left(\frac{x}{2}\right)=0$

$\frac{x}{2}=0$ $\frac{x}{2}=\pi$

$x=0$ $x=2\pi$

$\int_{0}^{2\pi}\sin\left(\frac{x}{2}\right)dx=-2\cos\left(\frac{x}{2}\right)\Big]_{0}^{2\pi}$

$=-2\cos(\pi)+2\cos 0$

$=-2(-1)+2\cos(1)$

$=4$

C 28. $f'(x)=\frac{1}{2}x+1$

$f(x)=\frac{1}{4}x^2+x+c$

$f(2)=\frac{1}{4}(2)^2+2+c=-3$

$3+c=-3$

$c=-6$

$f(x)=\frac{1}{4}x^2+x-6$

$f(4)=\frac{1}{4}(4)^2+4-6$

$=4+4-6$

$=2$

D 29. $g'(x)=4+\frac{f'(x)x-f(x)}{x^2}$

$g'(3)=4+\frac{f'(3)\cdot 3-f(3)}{9}$

$=4+\frac{4(3)+6}{9}$

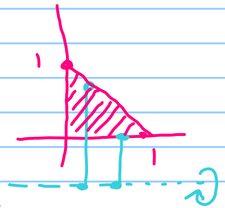
$=6$

$g(3)=4(3)+\frac{f(3)}{3}=12+\frac{-6}{3}$

$=10$

$y-10=6(x-3)$

C 30 $y=-x+1$



$\pi\int_{0}^{1}(-x+1+1)^2-(1)^2dx=$

$\pi\int_{0}^{1}(-x+2)^2-1\,dx=$

$\pi\int_{0}^{1}x^2-4x+4-1\,dx=$

$\pi\int_{0}^{1}x^2-4x+3\,dx=\pi\left[\frac{1}{3}x^3-2x^2+3x\right]_{0}^{1}=$

$\pi\left(\frac{1}{3}-2+3\right)=\pi\left(\frac{1}{3}+\frac{3}{3}\right)=\frac{4\pi}{3}$

$\int_0^1 f(x)dx < \int_0^2 f(x)dx$

B

31. $\int_0^2 f(x)dx > 0$

$\int_0^4 f(x)dx < 0$

$\int_0^5 f(x)dx < 0$

*compare signed areas

C 32. $x'(t) = v(t) = 4t^3 - 30t^2 + 58t - 36$

$|v(2)| = 8$

$|v(3)| = 24$

$|v(4)| = 28$

$|v(5)| = 4$

Speed $= |v(t)|$

D 33.

$f'(x) > 0 \rightarrow f(x)$ Inc

$f'(x)$ Inc on $(-\infty, 0) \rightarrow$ $f(x)$ c. up

$f'(x)$ Dec on $(0, \infty) \rightarrow$ $f(x)$ c. down

As $x \rightarrow \pm\infty$, $f'(x) \rightarrow 0 \rightarrow$ $f(x)$ approaches a H.A.

C 34. $f'(t) = e^{t\cos t}(\cos(t) - t\sin(t))$

A maximum occurs when $f'(t)$ changes from positive to negative or at an endpoint.

$f(0) = 0$

$f(.860) = .753$

* $f(6.437) = 577.827$

$f(10) = -1$

A 35.

$2 = 2e^{2t}$

$e^{2t} = 1$

$\ln 1 = 2t$

$t = 0$

$4 = 2e^{2t}$

$2 = e^{2t}$

$\ln 2 = 2t$

$t = \frac{1}{2}\ln 2$

$$\int_0^{\frac{1}{2}\ln 2} |2e^{2t}| dt = 1$$

B 36. $\Delta x = \frac{b-a}{n} = \frac{1}{n}$ $\therefore$ b-a=1

$$\lim_{n\to\infty} \underset{\uparrow \atop \text{width}}{\frac{1}{n}} \Big(\underset{\uparrow \atop \text{height}}{\left(\frac{1}{n}\right)^2} + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{n-1}{2}\right)^2 \Big)$$

$$\left(\frac{1}{n}\right)^2 = (\underset{\uparrow \atop \text{input}}{\Delta x})^2$$

$$\int_0^1 x^2 dx$$

B 37. $y'(t) = \frac{3}{4}\cos(3t) + \frac{3}{6}\sin(3t)$

$y''(t) = -\frac{9}{4}\sin(3t) + \frac{3}{2}\cos(3t)$

$y''(t)$ changes from positive (above x-axis) to negative (below x-axis) 4 times on $[0, 8]$

B 38. Area of Square = 4

Below: $\int_0^2 -x^2 + 2x\, dx = \frac{4}{3}$

Above: $4 - \frac{4}{3} = \frac{8}{3}$

Prob $= \frac{\frac{8}{3}}{4} = \frac{2}{3}$

C 39. $f'(x) = \sin^{-1}(x)$

$f'(.4) = \sin^{-1}(.4) \approx .412$

B 40. $f'(6) > 0$ b/c $f(x)$ Inc on $(-\infty, 8)$

* $f''(4) < 0$ b/c $f(x)$ is concave down on $(-\infty, 10)$

$f''(10) = 0$ b/c $x = 10$ is a P.O.I

$f''(12) > 0$ b/c $f(x)$ is concave up on $(10, \infty)$

D 41. $y = \sin x + \cos x \rightarrow \frac{dy}{dx} = \cos x - \sin x$

substitute + check

I. $\underbrace{\sin x + \cos x}_{y} + \underbrace{\cos x - \sin x}_{y'} \neq 2\sin x$

✓ II. $\underbrace{\sin x + \cos x}_{y} + \underbrace{\cos x - \sin x}_{y'} = 2\cos x$

✓ III. $\cos x - \sin x - \sin x - \cos x = -2\sin x$

D

42. g c. down to g. c up

$\therefore \underbrace{g'(x) \text{ dec to } g'(x) \text{ Inc}}_{\text{minimum of } g'(x)}$

C 43. $2y\frac{dy}{dx} - 3x^2 - 30x = 0$

$\frac{dy}{dx} = \frac{3x^2 + 30x}{2y} = 0$

$3x^2 + 30x = 0$

$3x(x + 10) = 0$

$x = 0 \quad x = -10$

D 44. Since g(x) is odd the signed areas on $[-a, 0]$ and $[0, a]$ are opposite in value so a-c are true.

C 45. $\text{avg} = \frac{1}{a-(-a)}\int_{-a}^{a} f(x)dx = 0$

$\therefore \int_{-a}^{a} f(x)dx = 0$

$\therefore f(x)$ is odd

1\. a) $s(0) = 0$

$s(t) = \int v(t)\,dt = -\cos(t) - e^{-t} + c$

$s(0) = -\cos(0) - e^{0} + c = 0$

$-1 - 1 + c = 0$

$c = 2$

$s(t) = -\cos(t) - e^{-t} + 2$

b) $v(t) = \sin(t) + e^{-t} = 0$

$t \approx 3.183$

c) $\frac{1}{5-0}\int_0^5 s(t)\,dt \approx 1.993$

d) $\int_0^5 |v(t)|\,dt \approx 4.206$

2\. a) At P, $d = 40$.

$10{,}000 = \frac{k}{40^2}$

$k = 16{,}000{,}000 = 1.6 \times 10^7$

b) $\cos\theta = \frac{40}{d} \rightarrow d = \frac{40}{\cos\theta}$

$L = \frac{k}{d^2} = \frac{k}{\left(\frac{40}{\cos\theta}\right)^2} = \frac{k}{\frac{1600}{\cos^2\theta}} = \frac{k\cos^2\theta}{1600}$

$L = \frac{(1.6)(10^7)\cos^2\theta}{(1.6)(10^3)}$

$L = 10^4\cos^2\theta$

c) $\frac{dL}{dt} = 2(10^4)\cos\theta(\sin\theta)\frac{d\theta}{dt}$

$= 2(10^4)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{\pi}{30}\right)$

$= -1{,}047.198$ Lumens/sec

The strength is decreasing at
1,047.198 lumens/sec

3\. a) $\int_1^5 1 - \frac{1}{x}\,dx = x - \ln|x|\Big]_1^5$

$= 5 - \ln 5 - 1 + \ln 1$

$= 4 - \ln 5$

b) $\pi\int_1^5 \left(2 - \frac{1}{x}\right)^2 - (2-1)^2\,dx$

c) $\int_1^5 2^x - 2\,dx =$

$\left[\frac{1}{\ln 2}2^x - 2x\right]_1^5 =$

$\frac{32}{\ln 2} - 10 - \frac{1}{\ln 2}\cdot 2 + 2 =$

$\frac{30}{\ln 2} - 8$

4\. a) $\frac{dh}{dt} = -k h^{1/2}$ $\quad h(0) = 16$

$\int h^{-1/2}\,dh = -\int k\,dt$

$2h^{1/2} = -kt + c \rightarrow 2\sqrt{h} = -kt + 8$

$\sqrt{h} = -\frac{k}{2}t + 4$

$2\sqrt{16} = 0 + c$

$8 = c$

$h(t) = \left(-\frac{k}{2}t + 4\right)^2$

b) $h(8) = 12.25$ $\quad h(8) = \left(-\frac{8k}{2} + 4\right)^2 = 12.25$

$-4k + 4 = 3.5$

$-4k = -\frac{1}{2}$

$k = \frac{1}{8}$

$h(t) = \left(-\frac{1}{16}t + 4\right)^2$

c) $h(t) = \left(-\frac{1}{16}t + 4\right)^2 = 0$

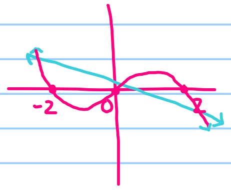
$-\frac{1}{16}t + 4 = 0$

$4 = \frac{1}{16}t$

$t = 64$ hours

5\. a) $m_c = \frac{15 - (-15)}{-3-3} = -5$

$y = 4x - x^3 = x(4 - x^2)$



$\frac{dy}{dx} = 4 - 3x^2 = -5$

$-3x^2 = -9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(\sqrt{3}, \sqrt{3})$ $\quad (-\sqrt{3}, -\sqrt{3})$

b) Chord: $y - 15 = -5(x+3)$

$y = -5x$

$D(x) = (4x - x^3) - (-5x) = -x^3 + 9x$

c) $D'(x) = -3x^2 + 9 = 0$

$3x^2 = 9$

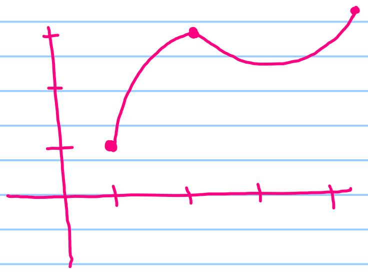
$x = \pm\sqrt{3}$ so $x = \sqrt{3}$ on $[0,3]$

$D''(x) = -6x$

$\left.\begin{array}{l} D'(\sqrt{3}) = 0 \\ D''(\sqrt{3}) < 0 \end{array}\right\}$ $x = \sqrt{3}$ yields a max value

$D(\sqrt{3}) = -(\sqrt{3})^3 + 9\sqrt{3} = -3\sqrt{3} + 9\sqrt{3} = 6\sqrt{3}$

d.



6\. a) $f(x)$ is Inc. on $(0,2) \cup (3,4)$ since $f'(x) > 0$.

b) $f(x)$ has a rel. minimum value at $x = 3$ since $f'(x)$ changes from negative to positive.

c) $f(x)$ has a rel. maximum value at $x = 2$ since $f'(x)$ changes from positive to negative.