

Practice Exam 3

MC - No calculator

B 1.  $A = \int_1^3 (3x^2 + 2x) dx$   
 $= x^3 + x^2 \Big|_1^3$   
 $= 27 + 9 - (1 + 1)$   
 $= \boxed{34}$

D 2.  $f(x) = \begin{cases} -x & \text{for } x < 0 \\ 0 & \text{for } x = 0 \\ x & \text{for } x > 0 \end{cases}$   
 $\int_{-4}^2 f(x) dx = \int_{-4}^0 -x dx + \int_0^2 x dx$   
 $= -\frac{x^2}{2} \Big|_{-4}^0 + \frac{x^2}{2} \Big|_0^2$   
 $= 0 - (-8) + 2 - 0$   
 $= \boxed{10}$

B 3.  $\int_0^b C(s) ds$   
 $\frac{\text{gal}}{\text{mile}} \cdot \frac{\text{mile}}{\text{hr}}$   
 $\boxed{\frac{\text{gal}}{\text{hr}}}$

D 4.  $\frac{dr}{dt} = 2 \frac{\text{in}}{\text{sec}}$   $\frac{dV}{dt} = ?$ ,  $r = 10 \text{ in}$

$\frac{d}{dt}(V = \frac{4}{3}\pi r^3)$   
 $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$   
 $\frac{dV}{dt} = 4\pi (10)^2 \cdot 2$   
 $= \boxed{800\pi \frac{\text{in}^3}{\text{sec}}}$

A 5.  $v(t) = t^2 - 4 \frac{\text{ft}}{\text{sec}}$

$\text{dist} = \int_1^3 |t^2 - 4| dt$   
 ~~$\int_1^3 (t^2 - 4) dt$~~   
 $\text{dist} = \int_1^2 (4 - t^2) dt + \int_2^3 (t^2 - 4) dt$   
 $= (4t - \frac{t^3}{3}) \Big|_1^2 + (\frac{t^3}{3} - 4t) \Big|_2^3$   
 $= \underbrace{8 - \frac{8}{3} - 4 + \frac{1}{3}}_{4 - \frac{7}{3} = \frac{5}{3}} + \underbrace{9 - 12 - \frac{8}{3} + 8}_{5 - \frac{8}{3} = \frac{7}{3}} + 8$   
 $= \frac{5}{3} + \frac{7}{3} = \frac{12}{3} = \boxed{4}$

D 6.  $\int_1^4 f'(x) dx$   
 $= f(4) - f(1)$   
 $= 2 - (-5)$   
 $= \boxed{7}$   
 1<sup>st</sup> fund Thm of Calculus!

A 7.  $\lim_{x \rightarrow 3} \frac{g(3) - g(x)}{3 - x} = -.628$   
 $\therefore g'(3) = -.628$   
 $g(x)$  is decreasing

C 8.  $\int_2^{10} \frac{x}{x^2+1} dx$   $u = x^2+1$ ,  $\frac{du}{dx} = 2x$   
 $dx = \frac{du}{2x}$   
 $x=1, u=2$   $x=3, u=10$   
 $= \frac{1}{2} \int_2^{10} \frac{du}{u}$   
 $= \frac{1}{2} \ln|u| \Big|_2^{10} = \frac{1}{2} (\ln 10 - \ln 2)$   
 $= \frac{1}{2} (\ln \frac{10}{2}) = \boxed{\frac{1}{2} \ln 5}$

A 9.  $\frac{d}{dx} \ln(\frac{1}{x^2-1}) =$   
 $= \frac{1}{x^2-1} \cdot -1(x^2-1)^{-2} \cdot 2x$   
 $= \frac{-2x(x^2-1)}{(x^2-1)^2}$   
 $= \frac{-2x}{x^2-1} = \boxed{\frac{2x}{1-x^2}}$

B 10.  $f(x) = e^{2x}$   
 $\frac{f(5) - f(0)}{5 - 0} = \frac{e^{10} - 1}{5}$

$f'(x) = 2e^{2x}$

$2e^{2x} = \frac{e^{10} - 1}{5}$

$e^{2x} = \frac{e^{10} - 1}{10}$

$2x = \ln(\frac{e^{10} - 1}{10}) \rightarrow x = \frac{1}{2} \ln(\frac{e^{10} - 1}{10})$   
 $0 < \frac{1}{2} \ln(\frac{e^{10} - 1}{10}) < 5$ ,  $\therefore$  one pt

B 11.  $f'(5) = -3$ ,  $f$  concave down  
 $f'(x)$  decr

a) 

x	f(x)
4	8
5	4
6	0

 $4 \rightarrow 8 > 4 \rightarrow 8 = -4$   
 $5 \rightarrow 4 > 0 \rightarrow 4 = -4$   
 $f'(x)$  const

b) 

x	f(x)
4	8
5	6
6	2

 $4 \rightarrow 8 > 6 \rightarrow 8 = -2$   
 $5 \rightarrow 6 > 2 \rightarrow 6 = -4$   
 $f'(x)$  decr

c) 

x	f(x)
4	8
5	6
6	5

 $4 \rightarrow 8 > 6 \rightarrow 8 = -2$   
 $5 \rightarrow 6 > 5 \rightarrow 6 = -1$   
 $f'(x)$  incr


d) 

x	f(x)
4	8
5	3
6	2

 $4 \rightarrow 8 > 3 \rightarrow 8 = -5$   
 $5 \rightarrow 3 > 2 \rightarrow 3 = -1$   
 $f'(x)$  incr

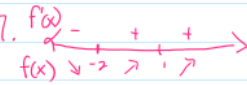
B 12.  $\lim_{t \rightarrow 0} \frac{\sqrt{1-2t} + t - 1}{t^2}$  "0/0"  
 $= \lim_{t \rightarrow 0} \frac{1}{2\sqrt{1-2t}} \cdot -2 + 1$   
 $= \lim_{t \rightarrow 0} \frac{-1}{\sqrt{1-2t}} + 1$  "0/0"  
 $= \lim_{t \rightarrow 0} \frac{1}{2(1-2t)^{3/2}} \cdot -2t$   
 $= \lim_{t \rightarrow 0} \frac{-1}{2(1-2t)^{3/2}} = \boxed{-\frac{1}{2}}$

D 13.  $\int_5^{12} f(x) dx$   
 $= 1(7) + 3(11) + 2(12) + 1(8)$   
 $= 7 + 33 + 24 + 8$   
 $= \boxed{72}$

B 14. VA  $x=2$   
 $\therefore \lim_{x \rightarrow 2} f(x) = \pm \infty$   
 $f'(x) > 0, x \neq 2$   
 Incr  
  
 $\lim_{x \rightarrow 2^-} f(x) = \infty$   
 $\lim_{x \rightarrow 2^+} f(x) = -\infty$

D 15.  $f(x)$  has min  
 when  $f'(x) < 0 \rightarrow f'(x) > 0$   
 at  $x=3$   
 Note: a)  $f'(5)=0$  identifies  
 Critical pt ONLY!  
 b) If  $f'(x) < 0 \rightarrow f'(x) > 0$   
 at  $x=5$ , min occurs at  
 $x=5!$   
 c) horiz tan line identifies  
 Critical pt. ONLY!

C 16.  $g(x) = \int_0^x f(t) dt$   
 $g'(x) = f(x)$   
 $g''(x) = f'(x)$   
 $g(x)$  changes from conc up  
 to conc down when  
 $g''(x) > 0 \rightarrow g''(x) < 0$   
 OR  
 $g'(x)$  incr  $\rightarrow g'(x)$  decr  
 This occurs at  $x=5$

A 17.   
 $f'(1) = 0$   
 Horiz tan line  
 $f'(x)$  decr  $(-\infty, 2)$   
 $f'(x)$  incr  $(-2, 1) \cup (1, \infty)$

B 18.  $\frac{d}{dx} \int_x^0 \frac{du}{1+u^2}$   
 $= \frac{d}{dx} \left( - \int_0^x \frac{du}{1+u^2} \right)$   
 $= \boxed{\frac{-1}{1+x^2}}$

D 19.  $g(x) = f(3x)$   
 $g'(x) = f'(3x) \cdot 3$   
 $g'(0.1) = 3f'(0.3)$   
 $g'(0.1) = 3(1.096)$   
 $\frac{1.096}{3} = 3.288$   
 $\frac{3.288}{3} = 1.096$

C 20.  $\frac{dy}{dx} = 4xy, (0,4)$   
 $\int \frac{dy}{y} = \int 4x dx$   
 $\ln|y| = 2x^2 + C$   
 $|y| = e^{2x^2 + C}$   
 $y = \pm e^{2x^2} e^C$   
 $y = C e^{2x^2}$   
 $4 = C e^0$   
 $C = 4$   
 $y = 4e^{2x^2}$

A 21.  $y = x^3 - 6x^2$   
 $y' = 3x^2 - 12x$   
 $y'' = 6x - 12 = 0$   
 $x = 2 \quad y'' < 0 \rightarrow y'' > 0$   
 $y(2) = 8 - 24 = -16$   
 $y'(2) = 12 - 24 = -12$   
 $(2, -16) \quad m = -12$   
 $y + 16 = -12(x - 2)$   
 $y + 16 = -12x + 24$   
 $y = -12x + 8$

B 22.  $\int_0^k \cos(ax) dx = \frac{1}{2}$   
 $\frac{1}{2} \sin(ax) \Big|_0^k = \frac{1}{2}$   
 $\frac{1}{2} [\sin 2k - \sin 0] = \frac{1}{2}$   
 $\frac{1}{2} \sin(2k) = \frac{1}{2}$   
 $\sin(2k) = 1$   
 $2k = \frac{\pi}{2}$   
 $k = \frac{\pi}{4}$

C 23. I.  $\int_a^b k f(x) dx = k \int_a^b f(x) dx, k \neq 0$   
 True  
 II.  $\int_a^b x f(x) dx = x \int_a^b f(x) dx$   
 False!  $x$  is NOT a scalar  
 III.  $\int_0^b f(x) dx = \int_0^c f(x) dx + \int_c^b f(x) dx$   
 True  
 I & III only

B 24.  $f(x) = \sqrt{e^{2x} + 1}$   
 $f'(x) = \frac{1}{2\sqrt{e^{2x} + 1}} \cdot 2e^{2x}$   
 $f'(0) = \frac{e^0}{\sqrt{e^0 + 1}}$   
 $= \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$

C 25.  $\frac{d}{dx}(x^2 + 3y^2 = 8 + 2xy)$

$$2x + 6y \frac{dy}{dx} = 2y + 2x \frac{dy}{dx}$$

$$(6y - 2x) \frac{dy}{dx} = 2y - 2x$$

$$\frac{d}{dx} \left( \frac{dy}{dx} = \frac{y-x}{3y-x} \right)$$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{dy}{dx} - 1\right)(3y-x) - \left(\frac{dy}{dx} - 1\right)(y-x)}{(3y-x)^2}$$

$$\left. \frac{dy}{dx} \right|_{(2,2)} = 0$$

$$\left. \frac{d^2y}{dx^2} \right|_{(2,2)} = \frac{(0-1)(6-2) - (0-1)(2-2)}{(6-2)^2}$$

$$= \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

A 26.  $x+2y=4$   
 $y = -\frac{1}{2}x + 2$

$$d = -\frac{1}{2}x + 2, r = -\frac{1}{4}x + 1$$

$$A = \frac{\pi}{2} r^2 = \frac{\pi}{2} \left( -\frac{1}{4}x + 1 \right)^2$$

$$V = \frac{\pi}{2} \int_0^4 \left( \frac{1}{16}x^2 - \frac{1}{2}x + 1 \right) dx$$

$$V = \frac{\pi}{2} \left( \frac{1}{48}x^3 - \frac{1}{4}x^2 + x \right)_0^4$$

$$V = \frac{\pi}{2} \left( \frac{1}{48}(64) - \frac{1}{4} \cdot 16 + 4 \right)$$

$$V = \frac{\pi}{2} \cdot \frac{4}{3} = \boxed{\frac{2\pi}{3}}$$

D 27.  $x(t) = (t+1)(t-3)^3$

$$v(t) = (t-3)^3 + 3(t-3)^2(t+1)$$

$$= (t-3)^2(t-3 + 3t+3)$$

$$= (t-3)^2(4t)$$

$$a(t) = 2(t-3)(4t) + 4(t-3)^2$$

$$= 4(t-3)(2t+t-3)$$

$$= 4(t-3)(3t-3) = 0$$

$$t=3, t=1$$

$$a'(t) \begin{matrix} + & - & + \\ \nearrow & \downarrow & \nearrow \end{matrix}$$

$$v(t) \text{ incr } (-\infty, 1) \cup (3, \infty)$$

A 28.  $\frac{1}{2}(\Delta x) [5 + 2f(18) + 2f(16) + 2f(12) + 7]$

$$\frac{1}{2}(\Delta x) [f(0) + 2f(6) + 2f(12) + 2f(18) + f(24)]$$

$$= 3(5 + 36 + 32 + 24 + 7)$$

$$= 3(104)$$

$$= \boxed{312}$$

D 29.  $\text{Avg } f = \frac{1}{7-3} \int_3^7 f(x) dx$

$$12 = \frac{1}{4} \int_3^7 f(x) dx$$

$$\boxed{48} = \int_3^7 f(x) dx$$

C 30.

$$L(x) > f(x)$$

when  $f(x)$  conc down  
or  $f''(x) < 0$

MC - Calculator Active

C 31.

$$V = \pi \int_0^\pi (\cos(\cos x))^2 dx$$

$$\boxed{V = 6.040}$$

A 32.  $h(x) = \frac{f(x)}{g(x)}$

$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$$

$$h'(3) = \frac{(-\frac{1}{3})(3) - (1)(1)}{(3)^2}$$

$$h'(3) = \boxed{-\frac{2}{9}}$$

A 33.

$$\int_0^c \cos x dx = \int_c^{\pi/2} \cos x dx$$

$$\sin x \Big|_0^c = \sin x \Big|_c^{\pi/2}$$

$$\sin c - \sin 0 = \sin \pi/2 - \sin c$$

$$2 \sin c = 1$$

$$\sin c = 1/2$$

$$\boxed{c = \pi/6}$$

D 34.  $f(x) = 5 + 15x + 6x^2 - x^3$   
 $f'(x) = 15 + 12x - 3x^2 = 0$   
 $x^2 - 4x - 5 = 0$   
 $(x-5)(x+1) = 0 \rightarrow x = -1, 5$

$f''(x) = 12 - 6x$   
 $f''(-1) = 18 > 0$   
 $f''(5) = -18 < 0$

Sign chart for  $f'(x)$ :  
 $f'(x) \begin{matrix} - & + & - \\ \downarrow & \uparrow & \downarrow \\ -1 & & 5 \end{matrix}$   
 decr on  $(-\infty, -1) \cup (5, \infty)$

A 35.  $\int_0^1 \frac{\tan^2 x}{x^2+1} dx = f(1) - f(0)$   
 $f(x) = \frac{1}{2} - \int_0^x \frac{\tan^2 t}{t^2+1} dt$   
 $f(0) = \frac{1}{2}$   
 $f(1) = \frac{1}{2} - \int_0^1 \frac{\tan^2 t}{t^2+1} dt$   
 $f(1) = \frac{1}{2} - \left(\frac{1}{2} - \frac{1}{2}\right) = \frac{1}{2}$   
 $f(1) = \frac{1}{2}$

B 36.  $f(x) = \frac{1}{4}x^2 - e^{-x} - \cos x - x$   
 $f'(x) = \frac{1}{2}x + e^{-x} + \sin x - 1$   
 $f''(x) = \frac{1}{2} - e^{-x} + \cos x$

Graph of  $f''(x)$  showing 6 points of inflection (POI) between  $x=0$  and  $x=20$ .

C 37.  $f(x) = 5 + 5.8 \sin\left(\frac{\pi x}{4}\right) - 15.7 \cos\left(\frac{\pi x}{3}\right)$   
 $f'(x) = \frac{5.8\pi}{4} \cos\left(\frac{\pi x}{4}\right) + \frac{15.7\pi}{3} \sin\left(\frac{\pi x}{3}\right)$

$f'(x)$  is max at  $x = 7.566$

Graph of  $f'(x)$  showing a peak at  $x = 7.566$ .

C 38.  $R(t) = 530e^{18t}$   
 Let  $v(t) = \# \text{ people w/ inf.}$   
 $\int_0^6 R(t) dt = v(6) - v(0)$   
 $v(6) = \int_0^6 R(t) dt + 725$   
 $v(6) = 645 \text{ people}$

D 39.  $f(x)$  is odd  $\rightarrow$  sym. origin  
 $\lim_{x \rightarrow -\infty} f(x) = -3$   
 then  $\lim_{x \rightarrow \infty} f(x) = 3$

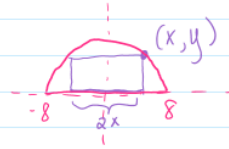
I  $\rightarrow$  true  
 II  $\rightarrow$  true - continuous  
 III  $\rightarrow$  true end behavior  
 $\lim_{x \rightarrow -\infty} f(x) = -3$  &  $\lim_{x \rightarrow \infty} f(x) = 3$

I, II and III true

C 40.  $f(x) = x^3 - 7x^2 + 25x - 39$   $g(x)$   
 $(c, 0)$   $(0, c)$   
 $0 = c^3 - 7c^2 + 25c - 39$   
 $c = 3$

$f'(x) = 3x^2 - 14x + 25$   
 $f'(c) = 10 \Rightarrow$  Then  $g'(c) = \frac{1}{10}$

A 41.  $\int_k^6 \frac{dx}{x+2} = \ln k, k > 0$   
 $\ln|x+2| \Big|_k^6 = \ln k$   
 $\ln(8) - \ln(k+2) = \ln k$   
 $\ln \frac{8}{k+2} = \ln k$   
 $\frac{8}{k+2} = k$   
 $k^2 + 2k = 8$   
 $k^2 + 2k - 8 = 0$   
 $(k+4)(k-2) = 0$   
 $k = -4$   $k = 2$

D 42. 

$y = \sqrt{64 - x^2}$   
 $A = 2xy$   
 $A = 2x\sqrt{64 - x^2}$   
 $A' = 2(\sqrt{64 - x^2}) + \frac{2x}{2\sqrt{64 - x^2}}$   
 $A' = 128 - 2x^2 - \frac{2x^2}{\sqrt{64 - x^2}} = 0$   
 $128 = 4x^2$   
 $32 = x^2$   
 $x = \pm\sqrt{32}$  length must be +  
 $x = \sqrt{32}$   $A' \begin{matrix} + & - \\ \uparrow & \downarrow \end{matrix}$   
 $\therefore x = \sqrt{32}$  max  
 $y = \sqrt{64 - 32} = \sqrt{32}$   
 $A = 2(\sqrt{32})(\sqrt{32}) = 64$

C 43.  $f(x) = \begin{cases} e^{-x} + 2 & \text{for } x < 0 \\ ax + b & \text{for } x \geq 0 \end{cases}$

If  $f$  is differentiable,  $f$  is cont.

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$e^0 + 2 = a(0) + b$$

$$3 = b$$

$$f'(x) = \begin{cases} -e^{-x} & \text{for } x < 0 \\ a & \text{for } x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f'(x) = \lim_{x \rightarrow 0^+} f'(x)$$

$$-e^0 = a$$

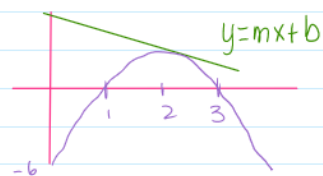
$$-1 = a$$

$$a + b = 3 - 1 = 2$$

B 44.  $\int_2^{10} R(t) dt = 731 \text{ gal}$

B 45.  $y = mx + b, b \geq 2$

$$f(x) = -2(x-2)^2 + 2$$



$$f'(x) = -4(x-2) = m, x \geq 2$$

At  $x=2, m=0$  and  $b=2$

Other extreme in Quad I

occurs at  $(3,0)$

$$f'(3) = -4$$

$$y = -4(x-3)$$

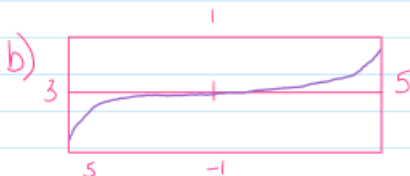
$$y = -4x + 12, b = 12$$

$$2 \leq b \leq 12$$

### Free Response - Calculator Active

1.  $v(t) = (t-4)^4 \sin(t-4)$

a)  $v(3) = -.841$   
 $S(3) = |v(3)| = .841$   
 $a(3) = v'(3) = 3.906$



c)  $v(t)$  is increasing on  $(3,5)$   
 since the graph is continuously rising in interval.

d)  $\text{dist} = \int_3^5 |v(t)| dt = \int_3^4 -v(t) dt + \int_4^5 v(t) dt$

$$\text{dist} = .293$$

2. a)  $\int_0^9 S(t) dt = 415.421 \text{ m}^3$

b)  $S(6) - R(6) = -39.909 \text{ m}^3/\text{hr}$

This is the rate of change of sand in the bin at  $t=6$  hrs. Since the rate is negative, more sand is being removed than added at this pt.

d)  $\int_0^9 S(t) dt - \int_0^9 R(t) dt = .00158 \text{ m}^3$   
 effectively empty

c)  $A(t) = \text{Net amt of sand in bin}$

$$A(t) = \int_0^t [S(x) - R(x)] dx$$

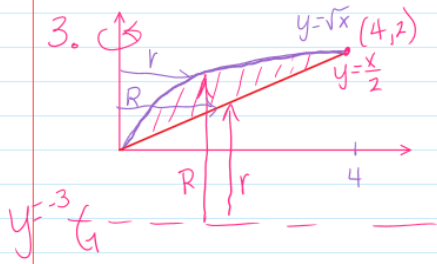
$$A'(t) = S(t) - R(t) = 0$$

$$S(t) = R(t) \text{ at } t = 3.837$$

On  $0 < t < 3.837$   $S(t) > R(t)$

On  $3.837 < t < 9$   $R(t) > S(t)$

Since sand was poured in at higher rate than it was removed until  $t = 3.837$  and is removed at higher rate than it is poured in after that, the amount of sand is a max at  $t = 3.837$  when  $S(t) = R(t)$ .



$$a) A = \int_0^4 \sqrt{x} - \frac{x}{2} dx = \left. \frac{2}{3}x^{3/2} - \frac{x^2}{4} \right|_0^4 = \frac{2}{3}(4)^{3/2} - \frac{4^2}{4} - 0$$

$$= \frac{16}{3} - \frac{12}{3} = \boxed{\frac{4}{3}}$$

$$b) R = 2y, \quad r = y^2$$

$$V = \pi \int_0^2 (2y)^2 - (y^2)^2 dy = \pi \int_0^2 (4y^2 - y^4) dy$$

$$V = \pi \left( \frac{4}{3}y^3 - \frac{y^5}{5} \right)_0^2 = \pi \left( \frac{32 \cdot 5}{3 \cdot 5} - \frac{32 \cdot 3}{5 \cdot 3} \right)$$

$$= \frac{32\pi}{15} (5-3) = \boxed{\frac{64\pi}{15}}$$

$$c) R = \sqrt{x} - (-3) = \sqrt{x} + 3$$

$$r = \frac{x}{2} - (-3) = \frac{x}{2} + 3$$

$$V = \pi \int_0^4 (\sqrt{x} + 3)^2 - \left( \frac{x}{2} + 3 \right)^2 dx$$

4. a)  $f'(a) = g'(b)$

$$f'(x) = 2x \quad g'(x) = 2(x-3)$$

$$f'(a) = 2a \quad g'(b) = 2b-6$$

$$2a = 2b-6$$

$$a = b-3$$

b)  $m = \frac{(b-3)^2 - (a^2-3)}{b-a}$       $b = a+3$

$$m = \frac{(a+3-3)^2 - a^2 + 3}{a+3-a} = \frac{a^2 - a^2 + 3}{3} = 1$$

$$\therefore f'(a) = 2a = 1 \quad \text{and} \quad g'(b) = 2b-6 = 1$$

$$a = \frac{1}{2} \quad b = \frac{7}{2}$$

c)  $m = 1$       $f(a) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 3$

$$= -\frac{11}{4}$$

$$y + \frac{11}{4} = x - \frac{1}{2}$$

$$y = x - \frac{13}{4}$$

$$d) A = \int_{\frac{1}{2}}^2 (x^2 - 3 - (x - \frac{13}{4})) dx + \int_2^{\frac{7}{2}} ((x-3)^2 - (x - \frac{13}{4})) dx$$

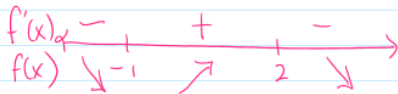
5.  $f(x) = k + 12x + 3x^2 - 2x^3$

a)  $f'(x) = 12 + 6x - 6x^2 = 0$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1$$



$f(x)$  is inc on  $(-1, 2)$   
since  $f'(x) > 0$

b) Rel max occurs at  $x = 2$  since  $f'(x) > 0 \rightarrow f'(x) < 0$

$$\therefore (2, 4)$$

$$\text{so } f(2) = 4 = k + 12(2) + 3(2)^2 - 2(2)^3$$

$$4 = k + 24 + 12 - 16$$

$$4 = k + 20$$

$$k = -16$$

c)  $f''(x) = 6 - 12x = 0$       $x = \frac{1}{2}$

$$\frac{f''(x)}{f(x)} \quad \begin{array}{c} + \quad - \\ \nearrow \quad \searrow \end{array}$$

$f(x)$  concave up on  $(-\infty, \frac{1}{2})$  since  $f''(x) > 0$

d) Rel min occurs at  $x = -1$  since  $f'(x) < 0 \rightarrow f'(x) > 0$

$$f(-1) = -16 + 12(-1) + 3(-1)^2 - 2(-1)^3$$

$$= -16 - 12 + 3 + 2$$

$$= -23$$

Rel min value =  $-23$

$$6. a) g(4.5) = \int_0^{4.5} f(t) dt = \frac{1}{2} \left( \frac{9}{2} + 3 \right) (3) + \frac{1}{2} \left( \frac{3}{2} \right) (3)$$

$$= \frac{3}{2} \left( \frac{15}{2} \right) + \frac{9}{4} = \frac{45}{4} + \frac{9}{4} = \frac{54}{4} = \frac{27}{2}$$

$$\begin{array}{r} 29 \\ 4 \overline{) 116} \\ \underline{8} \phantom{0} \\ 36 \end{array}$$

$$g'(4.5) = f(4.5) = 0$$

$$g''(4.5) = f'(4.5) = -2$$

$$b) \text{Av} f = \frac{1}{5 - (-3)} \int_{-3}^5 f(x) dx = \frac{1}{8} \left[ \frac{1}{2} \left( 6 + \frac{9}{2} \right) (3) + \frac{27}{2} - \frac{1}{2} \left( \frac{5}{2} \right) (1) + \frac{1}{2} (2) (1) \right]$$

$$= \frac{1}{8} \left[ \frac{3}{2} \left( \frac{21}{2} \right) + \frac{27}{2} - \frac{5}{4} + 1 \right] = \frac{1}{8} \left( \frac{116}{4} \right) = \frac{29}{8}$$

$\frac{6^3/4}{\phantom{+}} + \frac{5^4/4}{\phantom{+}} - \frac{5/4}{\phantom{+}} + \frac{4/4}{\phantom{+}}$

c) POI on  $g$  occur when  $g''(x)$  changes sign or  $g'(x)$  changes direction.  
 POI occurs at  $x=5$  where  $g''(x) < 0 \rightarrow g''(x) > 0$

Since  $f'(x) < 0 \rightarrow f'(x) > 0$

d)  $g'(x)$   $\left[ \begin{array}{c} + \\ \rightarrow \end{array} \right] \begin{array}{c} + \\ \rightarrow \end{array} \left[ \begin{array}{c} - \\ \rightarrow \end{array} \right] \begin{array}{c} + \\ \rightarrow \end{array} \left[ \begin{array}{c} + \\ \rightarrow \end{array} \right]$   
 $g(x)$   $\left[ \begin{array}{c} - \\ \rightarrow \end{array} \right] \begin{array}{c} - \\ \rightarrow \end{array} \left[ \begin{array}{c} - \\ \rightarrow \end{array} \right] \begin{array}{c} - \\ \rightarrow \end{array} \left[ \begin{array}{c} - \\ \rightarrow \end{array} \right]$   
 Rel max occurs at  $x=4.5$ ,  $g'(x) > 0 \rightarrow g'(x) < 0$

Also, consider endpoints.

$$g(-3) = \int_0^{-3} f(t) dt < 0 \Rightarrow g(x) \text{ increases from } g(-3) \text{ to } g(4.5) \rightarrow \text{No max at } g(-3).$$

$$g(4.5) = \frac{27}{2}$$

$$g(9) = \int_0^9 f(t) dt = \frac{27}{2} - \frac{5}{4} + 1 = \frac{54}{4} - \frac{5}{4} + \frac{4}{4} = \frac{53}{4}$$