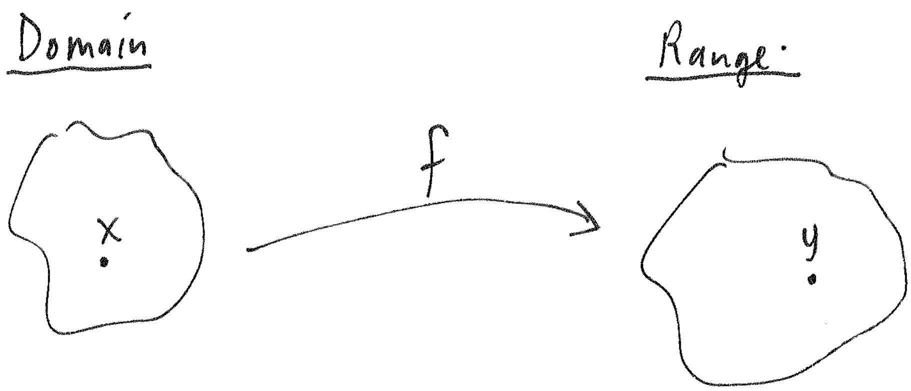
ch.2. : Review of key alg. concepts:

1. Functions

A *function* is a correspondence between two sets such that every member of the first set corresponds to *exactly one* member of the second.

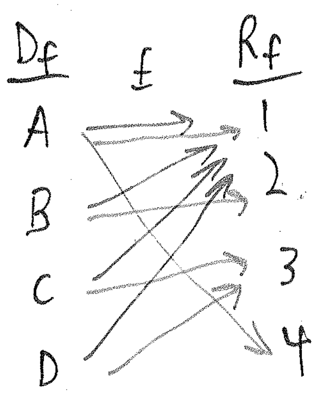


Correspondence

We write $y = f(x)$.

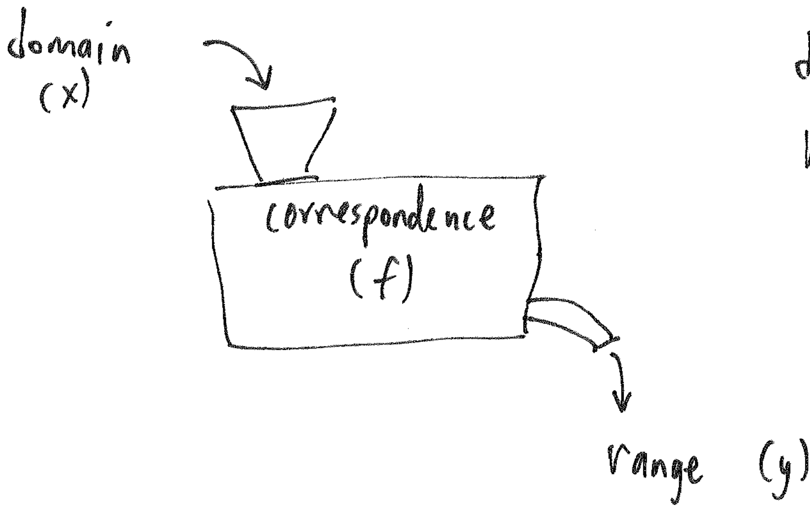
y is the unique member of the range that corresponds to x (from the domain).

Ex.



Yes
No

1.1 Functions as machines:

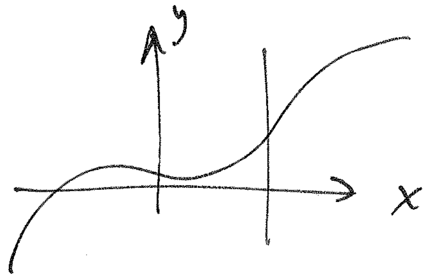


domain: inputs (x)

range: outputs (y)

1.2. Graphs of functions:

$y = f(x)$

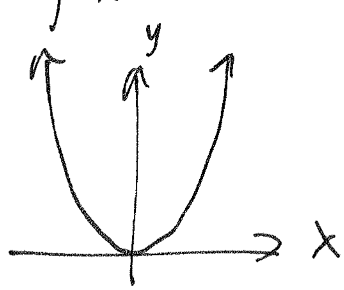


1.3 Vertical Line Test:

If any vertical line crosses a graph more than once, then the graph is not the graph of a function.

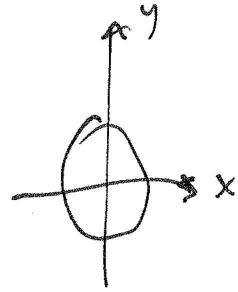
Exs.

$y = x^2$



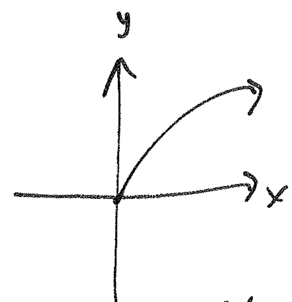
Yes.

$x^2 + y^2 = 1$



No.

$y = \sqrt{x}$



yes.

$y^2 = x$. No.

1.4. Domain: Find the domains of the functions.

Ex. $y = \sqrt{x-2}$

$f(x) = \dfrac{x+3}{x^2 - 7x + 12}$

Major restrictions: $\sqrt{\ }$, and $\div$ but also logs. (+bs)

1.5. Evaluating functions:

Ex. $f(x) = x^2 + 2x + 1$

$f(5) = 5^2 + 2(5) + 1$

$= 25 + 10 + 1$

$= 36$

$f(a) = a^2 + 2a + 1$

$f(x+h) = (x+h)^2 + 2(x+h) + 1$

$= (x+h)(x+h) + 2x + 2h + 1$

$= x^2 + 2xh + h^2 + 2x + 2h + 1$

Ex. $f(x) = x^5$

$f(2) = 2^5 = 32$

$f(x+h) =$ PASCAL'S TRIANGLE!

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$

1.6. Difference Quotients!

The essence of differential calculus (as we will soon see).

For any function $y=f(x)$, its difference quotient is

$$DQ(f) = \frac{f(x+h)-f(x)}{h} \quad \text{for } h \neq 0.$$

Ex. $f(x) = 3x+5$

$$f(x+h) = 3(x+h)+5$$
$$= 3x+3h+5$$

$$f(x+h)-f(x) = 3x+3h+5-(3x+5)$$
$$= \cancel{3x}+3h\cancel{+5}-\cancel{3x}-\cancel{5}$$
$$= 3h$$

$$DQ(f) = \frac{f(x+h)-f(x)}{h} = \frac{3\cancel{h}}{\cancel{h}} = \boxed{3}$$

Ex. $f(x) = x^2+2x+1$

$$f(x+h) = (x+h)^2+2(x+h)+1$$
$$= x^2+2xh+h^2+2x+2h+1$$

$$f(x+h)-f(x) = \cancel{x^2}+2xh+h^2+\cancel{2x}+2h\cancel{+1}-(\cancel{x^2}+\cancel{2x}\cancel{+1})$$
$$= 2xh+h^2+2h$$

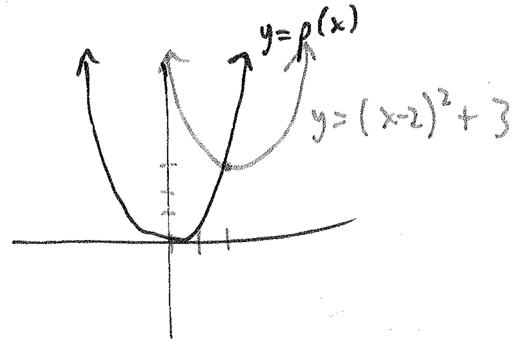
$$DQ(f) = \frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+2h}{h} = \frac{\cancel{h}(2x+h+2)}{\cancel{h}} = \boxed{2x+2+h}$$

1.7. <u>Transformations of graphs:</u>

Start with a known graph, called a parent function (or basic function) $y = p(x)$.

<u>Horizontal Translations</u> let $c > 0$

1. $y = f(x) = p(x+c)$: shift left c.
2. $f(x) = p(x-c)$: right c.
3. $f(x) = p(x) + c$: up c
4. $f(x) = p(x) - c$: down c.



<u>Ex</u>. $y = (x-2)^2 + 3$

$p(x) = x^2$

right 2, up 3.

<u>Ex</u>. $y = \sqrt{x+3} + 1$

<u>Ex</u>. $y = -|x| - 2$ $\leftarrow$!

$f(x) = -p(x)$ reflect over x-

$f(x) = p(-x)$ reflect over y-

2. Completing the Square

Most powerful tool for working with quadratic functions (parabolas).

2.1. Vertex of Parabola

Ex. $f(x) = x^2 + 4x + 9$

$b = 4$, $\frac{b}{2} = 2$, $\left(\frac{b}{2}\right)^2 = 4$

$$f(x) = (x^2 + 4x + 4) - 4 + 9$$
$$= (x+2)^2 + 5$$

vertex of parabola!

$$\boxed{V = (-2, 5)}$$

Standard equation of parabola: $f(x) = a(x-h)^2 + k$

(h,k) = vertex.

Ex. $f(x) = x^2 - 5x + \frac{1}{2}$

$b = -5$, $\left(\frac{b}{2}\right) = -\frac{5}{2}$, $\left(\frac{b}{2}\right)^2 = \frac{25}{4}$

$$f(x) = \left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{1}{2}$$
$$= \left(x - \frac{5}{2}\right)^2 - \frac{23}{4}$$

$$\boxed{V = \left(\frac{5}{2}, \frac{-23}{4}\right)}$$

Ex. $f(x) = -2x^2 + 16x - 4 = -2(\underline{x^2 - 8x + 2})$

$b = -8$, $\frac{b}{2} = -4$, $\left(\frac{b}{2}\right)^2 = 16$

$$f(x) = -2\left[(x^2 - 8x + 16) - 16 + 2\right]$$
$$= -2\left[(x-4)^2 - 14\right]$$
$$= -2(x-4)^2 + 28$$

$$\boxed{V = (4, 28)}$$

2.2. Solving eqn's using CtS.

Ex. Solve: $x^2 + 4x - 1 = 0$

$$(x+2)^2 - 5 = 0$$

$$(x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$\boxed{x = -2 \pm \sqrt{5}}$$

Ex. Solve: $9x^2 - 18x - 81 = 0$

$$9(x^2 - 2x - 9) = 0$$

$$(x-1)^2 - 10 = 0$$

$$(x-1)^2 = 10$$

$$x-1 = \pm\sqrt{10}$$

$$\boxed{x = +1 \pm \sqrt{10}}$$

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3. Logs and Exponentials

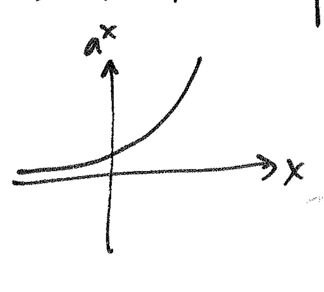
3.1. Exponential ~~Equations~~ (functions)

domain : $(-\infty, \infty)$

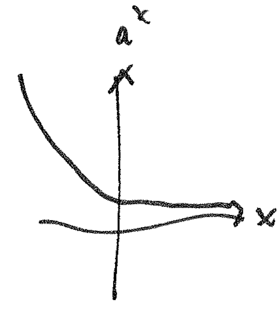
range : $(0, \infty)$

$f(x) = a^x$ for $a > 0$, $a \neq 1$.

the variable is in the exponent.



for $a > 1$



for $a < 1$.

3.2. Exponential Equations

Ex. $2^x = 32$ solve for x.

$\cancel{2}^x = \cancel{2}^5$

$\boxed{x=5}$

Ex. solve for x: $9^{x+2} = 3^x$

$(3^2)^{x+2} = 3^x$

$\cancel{3}^{2x+4} = \cancel{3}^x$

$2x+4 = x$

$\boxed{x=-4}$

3.3. Logarithms

Definition. $\log_a x = y$ means $a^y = x$ $a \neq 1,\ a > 0.$

Ex. $\log_2 8 = ?$

Write $\log_2 8 = y \Rightarrow 2^y = 8$

$2^y = 2^3$ so $\log_2 8 = 3.$

$\boxed{y=3}$

Logs are exponents.

$\log_{10} = \log$ = common log.

$$\begin{cases} \log_a 1 = 0 \\ \log_a a = 1 \end{cases}$$