Name: KEY

Business Calculus

Quiz 7

1. Use differentials or a tangent line approximation to estimate $\sqrt{25.1}$.

$f(a+dx) \approx f(a) + dy$, $dy = f'(a)dx$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

so $\sqrt{25.1} \approx \sqrt{25} + dy = 5 + 0.01$

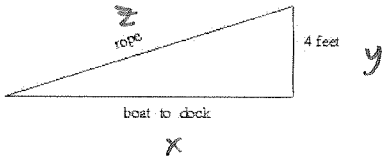
$= \boxed{5.01}$

$a = 25$

$dx = 0.1 = \frac{1}{10}$

$dy = f'(25) \cdot \frac{1}{10} = \frac{1}{2 \cdot 5} \cdot \frac{1}{10} = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} = 0.01$

2. A boat is being pulled toward a dock. If the rope is being pulled in at 3 feet per second, how fast is the distance between the dock and the boat decreasing when it is 30 feet from the dock?



$z^2 = x^2 + y^2$ $\quad x = 30,\ y = 4$ $\quad \frac{dy}{dt} = 0,\ \frac{dz}{dt} = 3,\ \frac{dx}{dt} = ?$

$\frac{d}{dt}\left[z^2 = x^2 + y^2\right] \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ (0)

$\Rightarrow \frac{dx}{dt} = \frac{z}{x}\frac{dz}{dt} = \frac{z}{30}(3)$

$z = \sqrt{30^2 + 4^2} = \sqrt{916} \Rightarrow \frac{dx}{dt} = \frac{\sqrt{916}}{30} \cdot 3 = \boxed{\frac{\sqrt{916}}{10} \text{ ft/sec}}$