Name: KEY

Business Calculus

Quiz 4

1. Let $f(x) = \dfrac{x^2+1}{x^2-1}$. Find each of the following:

a) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2+1}{x^2-1} = \dfrac{1}{1} = \boxed{1}$

b) $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \dfrac{x^2+1}{(x+1)(x-1)} = \dfrac{1^2+1}{(1+1)(1^- - 1)} = \dfrac{2}{2\cdot 0^-} = \boxed{-\infty}$

c) $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \dfrac{x^2+1}{(x+1)(x-1)} = \dfrac{1^2+1}{(1+1)(1^+ - 1)} = \dfrac{2}{2\cdot 0^+} = \boxed{+\infty}$

$\neq$

d) $\lim\limits_{x\to 1} f(x) =$ DNE

2. Use the *limit definition* of derivative to find $f'(x)$ for $f(x) = 2x^2 + 8$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

1. $f(x+h) = 2(x+h)^2 + 8 = 2(x^2+2xh+h^2)+8 = 2x^2+4xh+2h^2+8$

2. $f(x+h)-f(x) = \cancel{2x^2} + 4xh + 2h^2 \cancel{+8} - (\cancel{2x^2} \cancel{+8}) = 4xh + 2h^2$

3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2}{h} = \dfrac{\cancel{h}(4x+2h)}{\cancel{h}} = 4x+2h$

4. $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}(4x+2h) = 4x+2(0) = \boxed{4x = f'(x)}$