

1. Find (and simplify) the difference quotient for the function $f(x) = x^2 + 2x$.

$$DQ(f) = \frac{f(x+h) - f(x)}{h}$$

$$1. f(x+h) = (x+h)^2 + 2(x+h) \\ = x^2 + 2xh + h^2 + 2x + 2h$$

$$2. f(x+h) - f(x) = \cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - (\cancel{x^2} + \cancel{2x}) \\ = 2xh + h^2 + 2h$$

$$3. \frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 + 2h}{h} = \frac{h(2x + h + 2)}{h} = \boxed{2x + h + 2}$$

2. Write the function in vertex form by completing the square: $f(x) = 2x^2 + 16x + 6$.

$$f(x) = 2x^2 + 16x + 6 \Rightarrow \frac{f(x)}{2} = x^2 + 8x + 3$$

$$\left. \begin{array}{l} b = 8 \\ \frac{b}{2} = 4 \\ \left(\frac{b}{2}\right)^2 = 16 \end{array} \right\} \begin{array}{l} \frac{f(x)}{2} = (x^2 + 8x + 16) - 16 + 3 \\ \frac{f(x)}{2} = (x+4)^2 - 13 \end{array}$$

$$\Rightarrow \boxed{f(x) = 2(x+4)^2 - 26}$$

What is the vertex of the parabola?

$$\boxed{V = (-4, -26)}$$