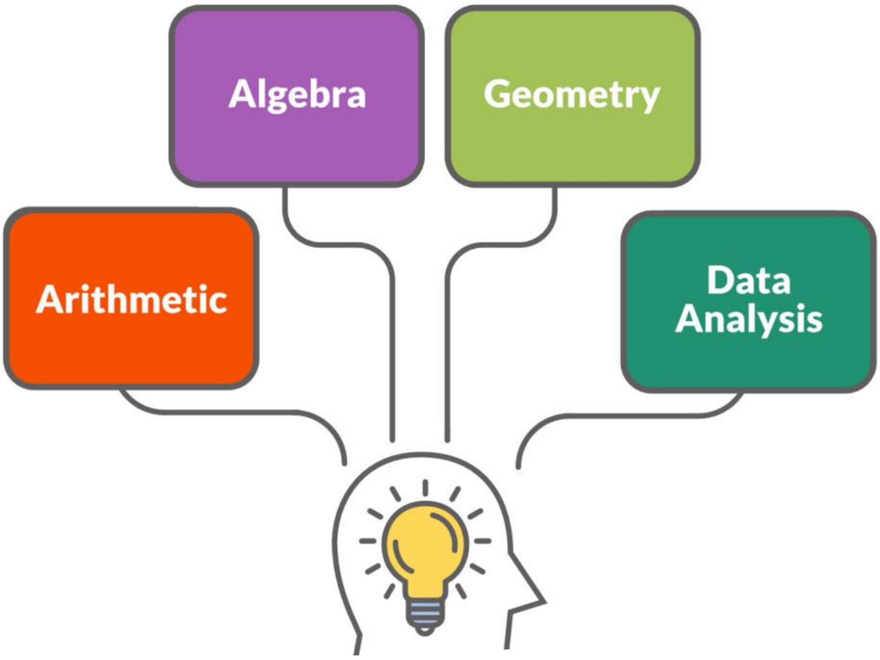
GRE MATH



Introduction

The Quantitative Reasoning section on the GRE might appear a bit scary to some test takers, especially because you have to correctly answer as many questions as possible within the time given to you. The key to performing well in the Math section is to make sure that:

- You have a strong understanding of the basics so that you are able to approach different questions with the right strategy.
- You remember the right formula to be used for a particular question - even though you may have strong understanding of the basics and you are capable of deriving the formula if necessary, remembering the formula by heart helps in saving precious time.

Arithmetic

Sets of Numbers

Integers

The set of integers include all the counting numbers ($1, 2, 3$ etc.) and their negative counterparts and the number 0. The integers can be represented as:

$$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$$

- Integers are ordered, i.e. given two integers a and b, then either $a > b$ or $a < b$ or $a = b$.
- The set of integers is not bounded above or below, i.e. there is neither a largest integer nor a smallest integer.
- Integers greater than zero are called positive integers and integers less than zero are called negative integers. (Note: 0 is neither positive nor negative!)
- The sum or product of any two integers is always an integer

Rational Numbers

Rational numbers are of the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$, p and q have no common factors besides the number 1, i.e. p and q are co-prime.

- The set of rational numbers includes all the integers
- Every fraction is a rational number
- The sum or product of any two rational numbers is always rational

Real Numbers and Irrational Numbers

The set of numbers that have a one-to-one correspondence with every point on the number line is the set of **Real numbers**.

Real numbers that are not rational numbers are called **irrational numbers**. $\sqrt{2}, \sqrt[3]{23}, \pi$, etc. are examples of irrational numbers.

Properties of Integers

Divisibility

An integer n is **divisible by** an integer k, if there exists another integer q such that $n = kq$.

- k is said to be **a *factor*** of n or we say $\boldsymbol{k}$ **divides** $\boldsymbol{n}$.
- n is said to be **a *multiple*** of k or we say $\boldsymbol{n}$ **is divisible by** $\boldsymbol{k}$.

For example, 72 is divisible by 18 because if $n = 72$ and $k = 18$, we can find an integer $q = 4$ such that $n = kq$ since $72 = 18 \times 4$.

- 18 is a factor of 72 or we say 18 divides 72.
- 72 is a multiple of 18 or we can say 72 is divisible by 18.

Some important properties of divisibility:

- All numbers are divisible by the number 1
- Every number divides itself
- No number is divisible by 0, but 0 is divisible by any number k
- If a divides b and a divides c, then a divides $(mb + nc)$ where m and n are any two integers.

- If a divides b and b divides c, then a divides c.

Tests for Divisibility

Divisibility by	**Test of divisibility**
2	If the units' digit of the number is divisible by 2, then the number is divisible by 2.
3	If the sum of the digits of the number is divisible by 3, then the number is divisible by 3. For example, the number 345 is divisible by 3 because the sum of the digits, $3 + 4 + 5 = 12$, is divisible by 3. However 248 is not divisible by 3 because the sum of the digits, 14, is not divisible by 3.
4	If the last two digits, taken as a single number, is divisible by 4, then the number is divisible by 4 For example, the number 2436 is divisible by 4 because the last two digits, 36 is divisible by 4.
5	If the units' digit is a 0 or a 5, then the number is divisible by 5
6	If a number is divisible by 2 and 3, then the number is divisible by 6 The number 345 is divisible by 3, but not divisible by 2. So we can conclude that it is not divisible by 6. However, 624 is divisible by both 2 and 3, and hence it is divisible by 6 as well.
7	This one is not straightforward! Separate the units' digit (say a_0) from the number and treat the remaining digits as a single number. From this number subtract $2a_0$ and check whether the resulting number is divisible by 7. If the resulting number is divisible by 7, then the original number is divisible by 7. For example, to check whether 392 is divisible by 7, calculate $39 - (2 \times 2)$ which evaluates to 35. Since 35 is divisible by 7, the number 392 is also divisible by 7.
8	If the last 3 digits, taken as single number, is divisible by 8 then the number is divisible by 8. For example, the number 166,064 is divisible by 8 because the last three digits, 064, is divisible by 8.

9	If the sum of all the digits number is divisible by 9, then the number is divisible by 9. For example, the number 786 is not divisible by 9 because the sum of the digits, $7 + 8 + 6 = 21$, is not divisible by 9. Whereas the number 684 must be divisible by 9 since the sum of the digits, $6 + 8 + 4 = 18$ is divisible by 9.
10	If the units' digit of the number is 0, then the number is divisible by 10
11	Take the sum of digits in the odd places and digits in the even places. If the difference between the sum is divisible by 11, then the number is divisible by 11. For example the number 10241 is divisible by 11, because $1 + 2 + 1 = 4$ and $0 + 4 = 4$, and the difference between the sums is 0 which is divisible by 11.
12	If a number is divisible by 3 and 4, then the number is divisible by 12
13	The method is similar to test for divisibility by 7. Separate the units' digit (say a_0) from the number and treat the remaining digits as a single number. From this number subtract $9a_0$ and check whether the resulting number is divisible by 13. If the resulting number is divisible by 13, then the original number is divisible by 13. For example, to check whether 832 is divisible by 13, do $83 - 9(2) = 65$. Since 65 is divisible by 13,832 is also divisible by 13.

Odd & Even Integers

Numbers that are divisible by 2 are called **even numbers** and numbers that leave a remainder 1 when divided by 2 are called **odd numbers**. Some properties of odd and even numbers:

- Even + Even = Even
- Odd + Even = Odd
- Odd +Odd =Even
- Even times Even = Even
- Odd times Odd = Odd
- Even times Odd = Even

Prime & Composite Numbers

- **A number is said to be prime** if it has exactly two factors – the number 1 and itself. Here's a list of the first few prime numbers:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, \ldots$$

- Numbers that have more than 2 factors are called **composite numbers.**
  Ex: 24 has 1,2,3,4,6,8,12,24 as factors .Since there are more than 2 factors , 24 is a composite number **32** has 1,2,4,8,16,32 as factors .So 32 is a composite number

- Note that 2 is the only even prime number.
- Note that the number 1 is neither prime nor composite – since it has only 1 factor.

Prime Factorization

The fundamental theorem of arithmetic states that every number can be represented uniquely as a product of primes. This representation is called the prime factorization of the number

For example the number 368 can be factorized as $368 = 2 \times 2 \times 2 \times 2 \times 23 = 2^4 \times 23$.

Factors of a Number

Consider a number n, whose prime factorization is given by
$n = (p_1)^{a_1} \times (p_2)^{a_2} \times (p_3)^{a_3} \times \ldots \times (p_m)^{a_m}$, where $p_1, p_2, p_3, \ldots, p_m$ are distinct primes.

- The number of prime factors of the number is equal to m
- The total number of factors of the number (including 1 and the number n itself) is given by
$$(1 + a_1)(1 + a_2)(1 + a_3) \ldots (1 + a_m)$$

For example, consider the number $n = 368$.

- The prime factorization of n is $= 2^4 \times 23^1$.
- The number of prime factors is just 2 ie., the primes 2 and 23.
- Comparing with the formula given above, for this case, $p_1 = 2, a_1 = 4$ and $p_2 = 23, a_2 = 1$
- Plugging in the values for a_1 and a_2 in the formula, we get the total number of factors to be
$$(1 + 4)(1 + 1) = 10.$$
- The factors of the number 368 are $1, 2, 4, 8, 16, 23, 46, 92, 184$ and 368.

GCD

The **GCD (Greatest Common Divisor)** of two positive integers a and b, is the largest number that is a factor of both a and b.

$$1 \leq GCD(a, b) \leq \min(a, b)$$

- If GCD of two numbers is 1, then the two numbers are said to be relatively prime. For example, the GCD of the numbers 8 and 15 is 1. So they are relatively prime.
- If GCD of two numbers is equal to the lesser of the two numbers, then the smaller number is a factor of the other.

LCM

The **LCM (Least Common Multiple)** of two positive integers a and b, is the smallest number that is a multiple of both a and b.

$$\max(a, b) \leq LCM(a, b) \leq ab$$

- If LCM of two numbers is equal to their product, then the two numbers are said to be relatively prime. For example, the numbers LCM of 8 and 15 is equal to 120 (which the product as well) – so they are relatively prime.
- If LCM of two numbers is equal to the larger of the two numbers, then the larger number is a multiple of the other.

Absolute Values & Inequalities

Absolute Value

The **absolute value** of a real number is defined as the distance of the point represented on the number line from the number 0.

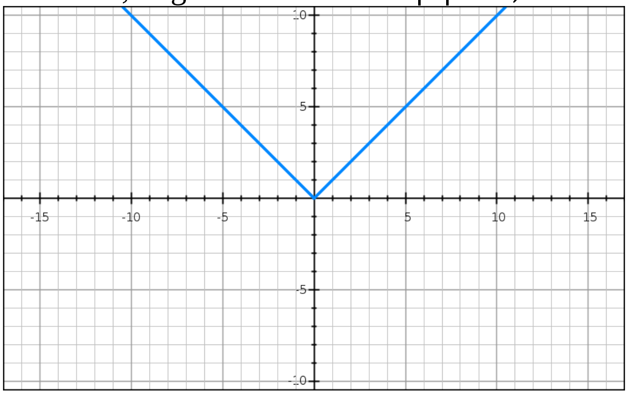
Even in real-life, absolute values are used while calculating distances. For example, if you were to travel from New York to Boston and then return from Boston to New York via the same route, then the distances would be the same in the both parts of the journey. i.e. it is not 'positive' in one direction and 'negative' in the return direction.

Note that distance is always positive, so the absolute value is always positive. Absolute value of x is denoted by $|x|$ which is also read as "modulus of x" or "mod x"

For example, the absolute value of the number 7 is 7 and that of the number (-2.43) is 2.43. In general,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Note that, in general if we have $|x| = a$, then x can either be $(+a)$ or $(-a)$.



That's the graph for $y = |x|$

Note that when x is negative , lets say $x = -5$, y is +5 .So when x is negative y is +ve and when x is +ve y is still +ve

Inequalities involving Absolute Values

Let's take an example to understand how to solve inequalities using absolute values.

Consider the inequality, $|x| \geq a$

The above inequality means, "the set of all x such that the distance of the x from the origin is greater than or equal to a units.

- Remember that the above set of values for x could occur on either side of the origin on the number line (the positive side or the negative side).
- The above inequality can be solved as follows:

$$|x| \geq a$$
$$\Rightarrow x \geq a \text{ or } x \leq -a$$

- So either x is greater than or equal to a or it is lesser than or equal to $-a$.

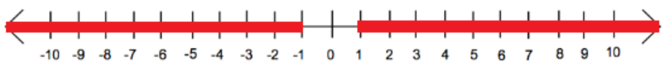
A picture is worth a thousand words Check the example graphs below to understand these inequalities better :

- Example :

$$|x| \geq 5$$

$$\Longrightarrow x \geq 5 \ \text{ or } x \leq -5$$

As you can see from the number line , the shaded region is to the left of -5 and to the right of which means x can take values from these parts of the number line only.



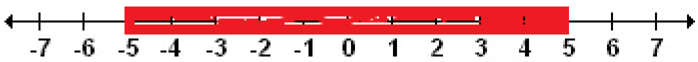
Let's now look at the other inequality with respect to the modulus

$$|x| \leq a$$
$$\Longrightarrow -a \leq x \leq a$$

Here is the example graph to understand this better .

$$|x| \leq 5$$
$$\Longrightarrow -5 \leq x \leq 5$$

- ⇨ X takes all values from -5 to 5 as is seen in the number line below



Properties of Exponents

Given a real number a, the following properties are always true for any two rational numbers m and n (in fact, these true even for real numbers, but that's not required for the GRE! ☺).

$$a^m \times a^n = a^{m+n}$$
$$(a^m)^n = a^{mn}$$

For example,

$$2^3 \times 2^4 = 8 \times 16 = 128$$
$$2^{(3+4)} = 2^7 = 128$$

Since both quantities are equal to 128, we can see than $2^3 \times 2^4 = 2^{3+4}$

Similarly,

$$(3^3)^2 = 27^2 = 729$$
$$3^{3\times 2} = 3^6 = 729$$

Since both quantities are equal to 729, we can see than $(3^3)^2 = 3^{3\times 2}$

Points to remember:

- $a^0 = 1$ for all real values of a (NOTE: 0^0 is controversial, but the consensus amongst mathematicians is that it is to be defined as 1)
- $1^x = 1$, for all finite real values of x
- $a^{-1} = \frac{1}{a}$ for all non-zero real values of a
- $a^{\frac{1}{2}} = \sqrt{a}$ and in general $a^{\frac{1}{n}} = \sqrt[n]{a}$ for all positive integers n.

For example ,
$\sqrt[3]{8}$=$8^{\frac{1}{3}} = 2$

Fractions & Decimals

- A rational number or a fraction when converted to a decimal will always be a terminating decimal or a non-terminating but recurring decimal.
  - If the denominator of a fraction <u>in its simplest form</u> has only 2 or 5 as its prime factors, then decimal form of the fraction will be a terminating decimal.
    - For example the fraction $\frac{17}{80}$ when converted to decimal form will be a terminating decimal, because the only prime factors of 80 are 2 and 5. $(80 = 2^4 \times 5)$
  - If the denominator of a fraction in its simplest form has any other prime factor besides 2 and 5, then the decimal form of the fraction will be non-terminating but recurring decimal.
    - For example, we can conclude that the fraction $\frac{37}{102}$ will not be a terminating decimal as the denominator 102 has $2, 3$ and 17 as prime factors. $(102 = 2 \times 3 \times 17)$

  Observe that these decimals are recurring ,
  ex: $\frac{32}{102} = 0.\overline{31372549019}$
  Also , $\frac{1}{9} = 0.\overline{1}$
  As you can see , the recurring digits are represented by a bar on top called as "$\boldsymbol{Vinculum}$".
  Also , the number of recurring digits in a recurring decimal say $\frac{1}{n}$ is atmost $n - 1$

- An irrational number when represented in a decimal form will have non-terminating and non-recurring decimal expansion. For example, the decimal expansion of $\sqrt{2} = 1.4142135623$ ... is non-terminating and there is no pattern of digits repeating.

Ratio and Proportion

- A ratio is a comparison or relationship between two or more numbers of the same kind , expressed as "to " or by the symbol colon ":"
    - For example , if there is a recipe for a cake that uses 2 cups of dough and 3 cups of milk for every cup of sugar, then "The ratio of dough to milk to sugar is $2:3:1$"

- The ratio of two quantities can also be represented as a fraction.

- If an increase in quantity results in a "proportionate" increase in another quantity, then the two quantities are said to be in "**Direct proportion**".
    - For example, the price and the number of items are directly proportional to each other.
- If an increase in one quantity results in "proportionate" decrease in another quantity , then the two quantities are said to be in "**Indirect or Inverse proportion**"
    - For example, increasing the speed to cover a particular distance will proportionately decrease the time to cover the distance.

Percentages

- Expressing number or ratio as a fraction of 100 is termed as "**Percentage**"
- To convert a ratio or fraction to percent , multiply the fraction by 100 and write it as a percent denoted by the symbol "%"
    - For example, $\frac{3}{4}$ is 75% as, $\frac{3}{4} \times 100 = 75$.

Algebra

Algebraic Identities

Identities of the form $(x + y)^n$

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
- $(x + y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + \cdots + {}^nC_ny^n$

(NOTE: The symbol ${}^nC_r$ represents the number of ways of choosing r objects from n distinct objects. The value can be evaluated using the formula, ${}^nC_r = \frac{n!}{(n-r)!r!}$

If you notice the above identities, it is easy to identify the pattern of decreasing powers of x and increasing powers of y in each of the terms. But, the values of the coefficients, do not seem to be in any obvious pattern. Here is a visual way to remember the coefficients by making use of Pascal's Triangle.

1	row 0
1 1	row 1
1 2 1	row 2
1 3 3 1	row 3
1 4 6 4 1	row 4
1 5 10 10 5 1	row 5
1 6 15 20 15 6 1	row 6
1 7 21 35 35 21 7 1	row 7

To find the coefficients of $(x + y)^n$, look at row number n. For example to find the coefficients of $(x + y)^4$ look at row number 4 and find the them to be $1, 4, 6, 4$ and 1. Now you can write down the expansion as:

$$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Here is a link to an animated gif which shows how to construct the Pascal's triangle.

Identities of the form $(x - y)^n$

- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- $(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
- $(x - y)^n = {}^nC_0x^n - {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 - \cdots + (-1)^n\,{}^nC_ny^n$

Identities of the form $x^n - y^n$

- $x^2 - y^2 = (x - y)(x + y)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^4 - y^4 = (x - y)(x^3 + x^2y + xy^2 + y^3)$

In general, for any natural number n,

- $x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots + y^{n-1})$

Identities of the form $x^n + y^n$

- $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
- $x^5 + y^5 = (x+y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$

In general, for any natural number n ,

- If n is odd, $x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots + y^{n-1})$

Linear Equations and Inequalities

Linear Equation in 1 variable

A linear equation is of the form $ax + b = 0$, where a and b are constants, and a is not 0. To solve this equation, we need to isolate the x on one side. We first subtract b from both sides – note that subtracting equal values from both sides of the equation will not affect the validity of the equation.

$$ax + b - b = 0 - b$$
$$\Rightarrow ax = -b$$

Similarly, dividing both sides of an equation by a non-zero number will not affect the validity of the equation. So let's divide both sides of the above equation by a

$$\frac{ax}{a} = \frac{-b}{a} \Rightarrow x = -\frac{b}{a}$$
$$\frac{ax}{a}$$

So the solution for the equation $ax + b = 0$ is given by $x = -\frac{b}{a}$.

Linear Inequality in 1 variable

When dealing with linear inequalities, the sign of a needs to be taken into account. For example, to solve the linear inequality $ax + b > 0$

- If a is positive, then the dividing both sides by a <u>will not</u> alter the direction of the inequality.

$$ax + b > 0$$
$$\Rightarrow ax > -b$$
$$\Rightarrow x >= -\frac{b}{a} \text{ (dividing both sides by } a\text{)}$$

- If a is negative, then dividing both sides by a <u>will</u> alter the direction of the inequality.

$$ax + b > 0$$
$$\Rightarrow ax > -b$$
$$\Rightarrow x < -\frac{b}{a} \text{ (dividing both sides by } a\text{)}$$

Pair of Linear Equations in 2 variables

Linear equations in two variables can be solved using Elimination or Substitution methods. Consider the following problem:

Ruth has 1 candy more than Sandra. If three times the number of candies that Ruth has is 6 more than twice the number of candies that Sandra has, then find the number of candies that each one of them have.

To solve the above problem, let's frame the equations. Let x be the number of candies that Ruth has, and y be the number of candies that Sandra has. Based on the first piece of information in the question, since Ruth has 1 candy more than Sandra, we can write in the form of an equation as $x - y = 1$.

Now we are also given that "3 times the number of candies that Ruth has is 6 more than twice the number of candies that Sandra has". Three times the number of candies that Ruth has is $3x$, whereas 6 more than twice the number of candies that Sandra has is $2y + 6$. Since these two quantities are given to be equal, we have $3x = 2y + 6$, which can be simplified as $3x - 2y = 6$.

So we have to solve the following two equations, to find the values of x and y.

$$3x - 2y = 6; x - y = 1$$

- **Elimination Method**
  - Multiply the second equation by 2 to get $2x - 2y = 2$
  - Subtract the above from the first equation
    $$(3x - 2y) - (2x - 2y) = 6 - 2$$
    $$\Longrightarrow x = 4$$
  - Since $x = 4$ and $x - y = 1$, y must be 3.
  - So the solution to the above pair of equations is $x = 4, y = 3$, i.e. Ruth has 4 candies and Sandra has 3 candies.
- **Substitution Method**
  - Rewrite the second equation as $x = 1 + y$.
  - Substitute this expression for x in the first equation to get
    $$3(1 + y) - 2y = 6$$
  - Simplify the above equation to get $y = 3$.
  - Substitute the value for y in the expression for x to get $x = 4$.
  - So the solution to the above pair of equations is $x = 4, y = 3$ which is the same as what we got using the previous method.

Quadratic Equations and Inequalities

Standard form of a Quadratic Equation

- The standard form of a quadratic equation is $ax^2 + bx + c = 0$, with $a \neq 0$. These equations can be solved either using splitting the middle term or using the quadratic formula.

- The solutions of a quadratic equation are called roots of the equation.

Solution of Quadratic Equations

Consider the quadratic equation $2x^2 - 7x + 3 = 0$. This equation can solved in two ways:

- **By splitting the middle term**
  - Consider the product of the coefficient of x^2 (i.e. a) and the constant term (i.e. c) {in this case it is $2 \times 3 = 6$
  - Find a pair of numbers, which on multiplication will yield the above result and on addition will be equal to the coefficient of x (i.e. b).
  - In this case, the numbers should have a product 6 and should add up to (-7). (-1) and (-6) satisfy this condition.
  - So we split the middle term as and solve as follows:

$$2x^2 - 7x + 3 = 0$$
$\Longrightarrow 2x^2 - x - 6x + 3 = 0$
$\Longrightarrow x(2x-1) - 3(2x-1) = 0$
$\Longrightarrow (x-3)(2x-1) = 0$
$\Longrightarrow (x-3) = 0$ or $(2x-1) = 0$ (Since product of two terms is 0, either one of them should be 0)
$\Longrightarrow x = 3$ or $x = \frac{1}{2}$

- **Using the quadratic formula**
  - For a general quadratic equation of the form $ax^2 + bx + c = 0$, with $a \neq 0$, is given by,
  $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
  - Substituting for a, b and c in the above equation we get
  $$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2}$$
  $$x = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$$
  $$x = 3 \text{ or } x = \frac{1}{2}$$

Quadratic Inequalities
Given a quadratic inequality of the form $ax^2 + bx + c < 0$, with $a \neq 0$, the solution for x can be found out using the following method:
- Solve the quadratic equation $ax^2 + bx + c = 0$ to find the roots. Say p and q are the roots of the quadratic equation, with $p < q$
- Take any real number in between p and q and check if the value satisfies the inequality.
- If it satisfies the inequality, then solutions is the set of all x such that, $p < x < q$.
- If the value does not satisfy the inequality, then the solution is the set of all x such that, $x < p$ or $x > q$.

For example, to solve the inequality $2x^2 - 7x + 3 < 0$:
- We find the roots of $2x^2 - 7x + 3 = 0$ which are $\frac{1}{2}$ and 3.
- Let's take a real number between $\frac{1}{2}$ and 3, say 1 and substitute $x = 1$ and check if the inequality is valid. Substituting $x = 1$, in $2x^2 - 7x + 3$ we get $2 \times (1^2) - 7 \times 1 + 3 = -2 < 0$, i.e. the inequality is true.
- So the set of all x such that $\frac{1}{2} < x < 3$, will satisfy the inequality and is the solution set of the given inequality.

Coordinate Geometry

Quadrants
The horizontal x axis and the vertical y axis intersect at the origin. These two axes divide the Cartesian plane into 4 quadrants. For a point P in the plane with coordinates (x, y), the values of x and y take different signs based on where the point lies:
- If P is on the x axis, then $y = 0$
- If P is on the y axis, then $x = 0$
- If P is in the 1st quadrant, then x and y are positive
- If P is in the 2nd quadrant, then x is negative and y is positive

- If P is in the 3rd quadrant, then x and y are negative
- If P is in the 4th quadrant, then x is positive and y is negative

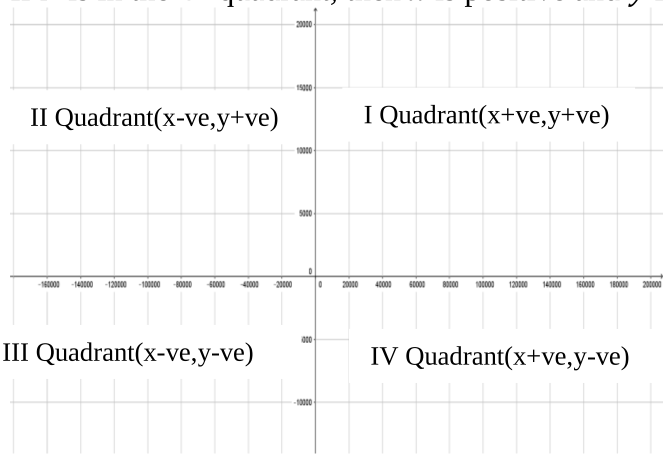


Fig:1

Distance Formula

The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

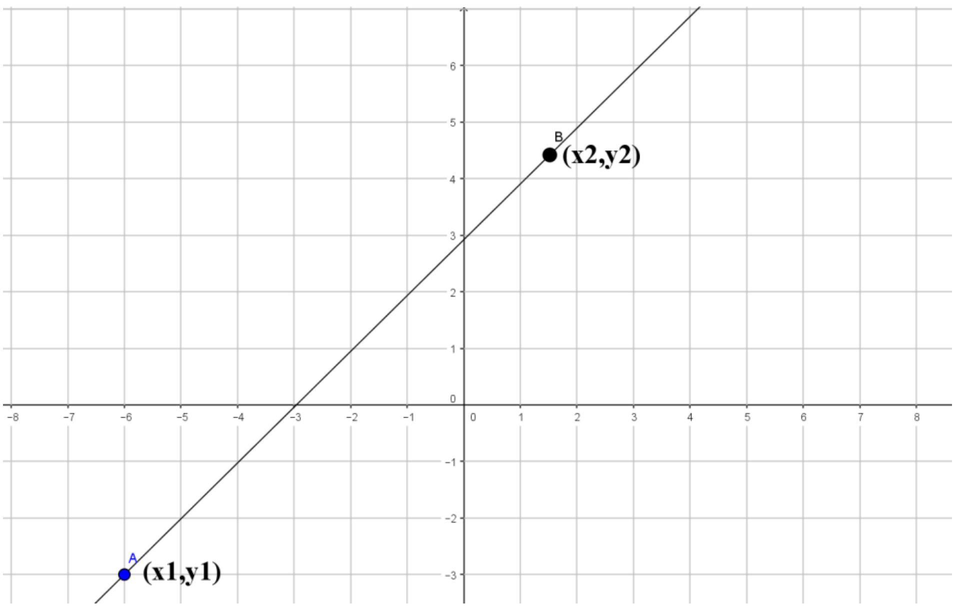


Fig. 2

Straight Lines

Any linear equation in two variables represents a straight line on the coordinate plane. So the general form of straight line is $ax + by + c = 0$.

The x-intercept of the line is x coordinate of the point where the line intersects the x axis. The x-intercept can be found out by substituting $y = 0$ in the equation of the line and finding the value of x.

Similarly, the y-intercept of the line is y coordinate of the point where the line intersects the y axis. The y-intercept can be found out by substituting $x = 0$ in the equation of the line and finding the value of y.

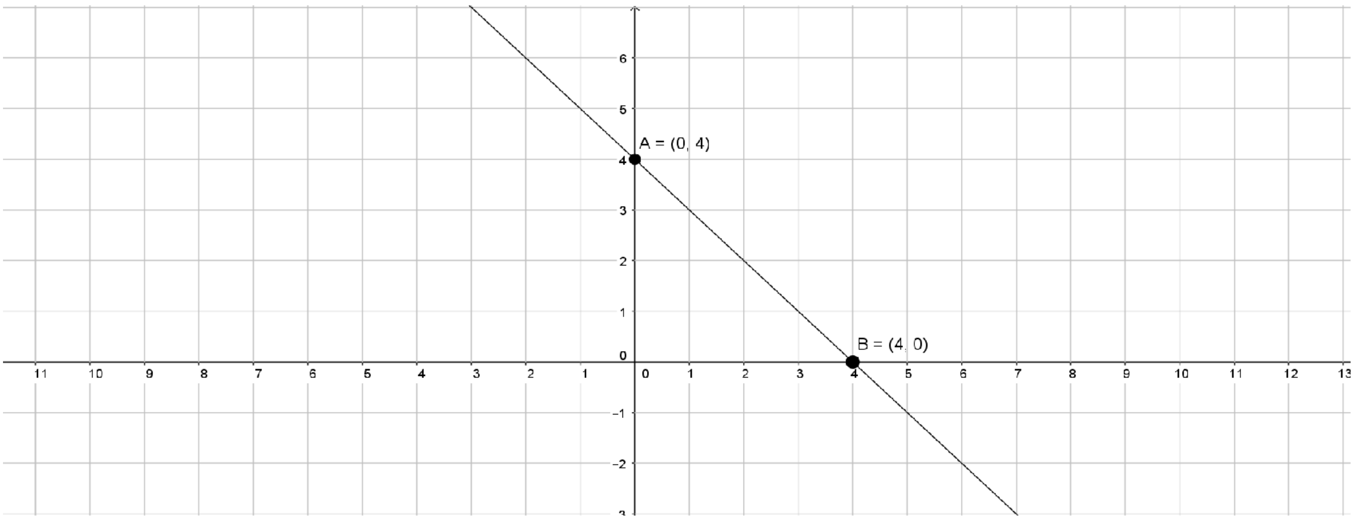


Fig. 3

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points on a line l, then the slope of the line, m, can be defined as

$$m = \frac{\text{Difference in } y\text{-coordinates}}{\text{Difference in } x\text{- coordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

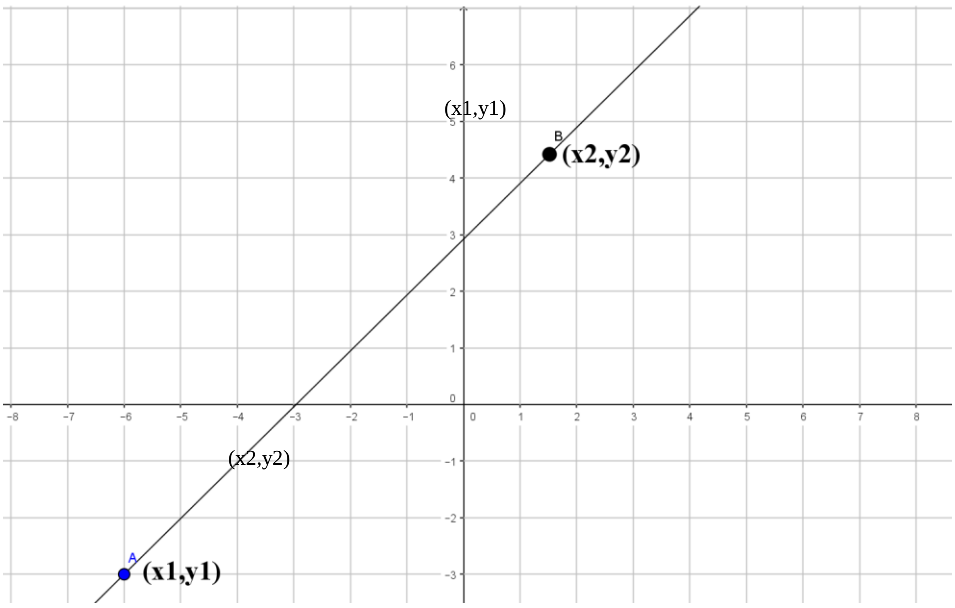


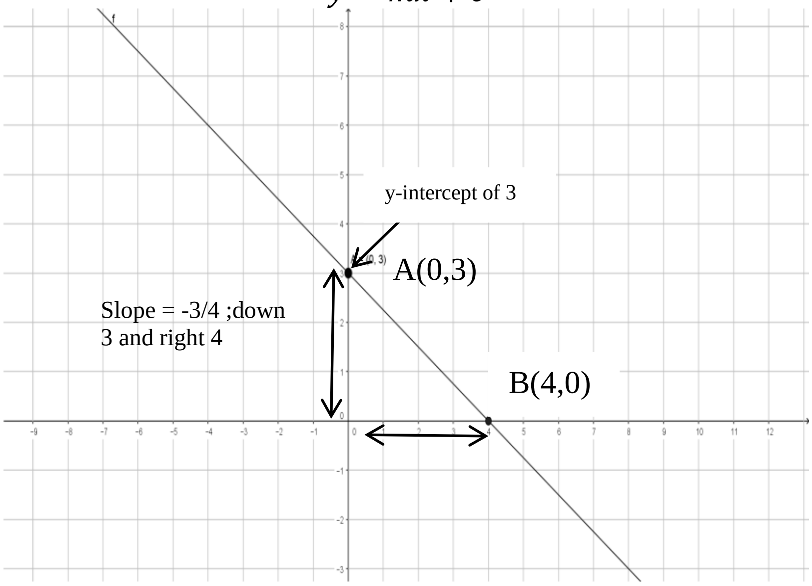
Fig.4

Note that the above definition of slope will only work for straight lines that are not parallel to the y-axis, i.e. lines that are not vertical. For vertical lines, there is no definition of slope.

Also note that for a horizontal line (i.e. lines parallel to the x-axis) the slope is always 0.

The equation of the straight line can also be written in terms of the slope, m, and the y-intercept, c as follows:

$$y = mx + c$$



Equation of line: $\mathbf{4y + 3x = 12 \Rightarrow y = -\frac{3}{4}x + 3 \Rightarrow m = -\frac{3}{4}}$ **and** $\mathbf{c = 3}$

Functions

A function can be thought of as a system that takes an input (or many inputs) and produces an output. For example, a function f can be defined as $f(x) = 2x + 5$. Given an input value for x, the function value is $f(x)$ which is also known as the value of f at x.

The set of all permissible values of x for which f is defined is called the domain of f. The set of all values of f is called the range of f.

Important Functions and their graphs

Linear Function: $f(x) = ax + b$, where a and b are constants and a is non-zero.

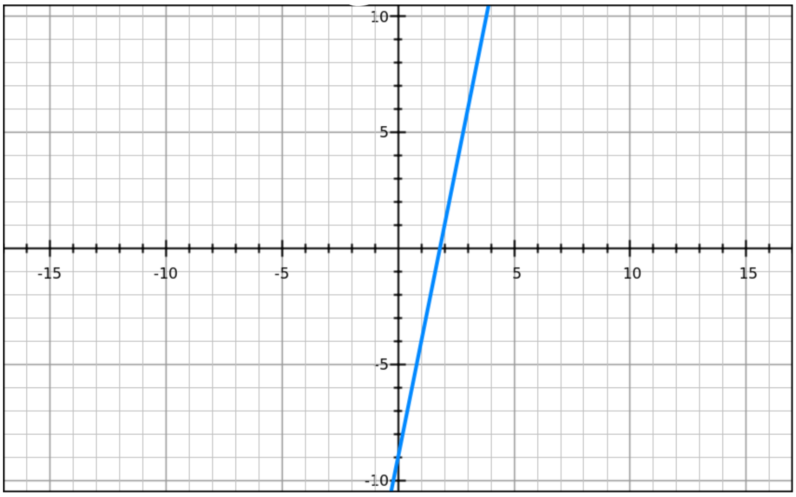


Fig 6-Linear function given by $\boldsymbol{f(x) = 5x - 9}$

Quadratic Function: $f(x) = ax^2 + bx + c$, where a, b and c are constants and a is non-zero.

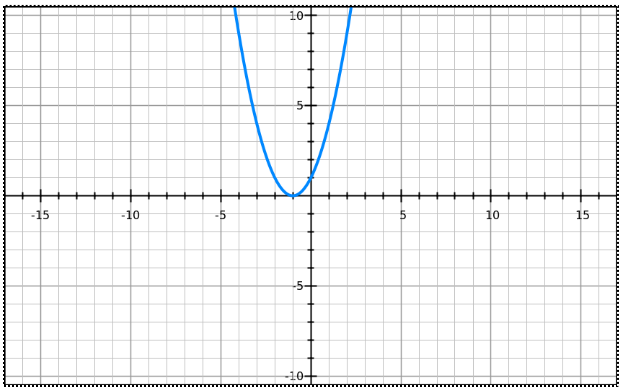


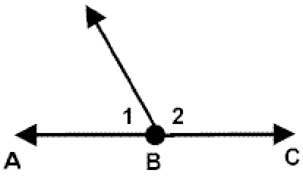
Fig 7-Quadratic function $\boldsymbol{f(x) = x^2 + 2x + 1}$

Note that the graph of all quadratic functions are parabolas.

Geometry

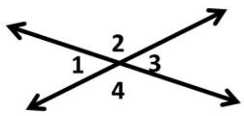
Lines and Angles

Linear Pair of Angles



In the above figure, angles 1 and 2 are adjacent and ABC is a straight line. So these two angles are called a linear pair of angles and they add up to 180° (i.e. they are supplementary)

Vertically Opposite Angles



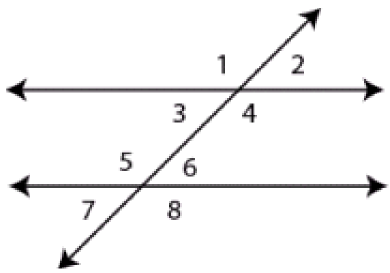
Angles 1 and 3 and Angles 2 and 4 are pairs of vertical angles.

When two lines intersect at a point, there are 4 angles that are formed, as shown in the figure above.

- Angles 1 and 3 form a pair of vertically opposite angles and they are equal to each other.
- Angles 2 and 4 form another pair of vertically opposite angles and they are also equal.

Parallel Lines and a Transversal

When two parallel lines are intersected by a third line (called a transversal) 8 angles are formed as shown in the figure below:



- Angles 1 and 5, angles 2 and 6, angles 3 and 7, angles 4 and 8 are all called corresponding pairs of angles. Corresponding angles are equal.
- Angles 3 and 5, angles 4 and 6 are called pairs of interior angles on the same side of the transversal. Interior angles on the same side are supplementary, i.e. they add up to 180°.
- Angles 3 and 6, angles 4 and 5 are called pairs of interior angles on the alternate sides of the transversal or alternate interior angles. Alternate interior angles are equal.
- Using the above three conditions, we have $\angle 1 = \angle 3 = \angle 5 = \angle 7$ and $\angle 2 = \angle 4 = \angle 6 = \angle 8$.

Note: The above conditions are actually "if and only if" conditions, i.e. their converses are also true. For example, if two lines are intersected by a third line and if one pair of corresponding angles are equal then we can conclude that the two lines are parallel.

Triangles

Triangles are 3-sided polygons. Triangles can be classified based on the length of their sides and based on their angles .

Classification based on sides:

- Scalene triangle – the lengths of all three sides are different
- Isosceles triangle –The lengths of two sides are equal.
    - The angles opposite to equal sides are equal
    - In an isosceles triangle , the altitude to the unequal side is also the median of the unequal side ie.it is the perpendicular bisector of the unequal side and also the angle bisector of the vertical angle from which it is dropped.
- Equilateral Triangle-The lengths of all three sides are equal .So each angle = 60 degrees.
    - The altitudes, medians , angle bisectors coincide in an equilateral triangle.
    - Among all triangles with the same perimeter , the triangle with the largest area will be equilateral

Classification based on angles:

- Right Triangle –One angle in 90 degrees
  The side opposite to the right angle is the longest side called Hypotenuse in a right triangle.
- Acute angled triangle-All three angles are less than 90 degrees
- Obtuse angled triangle –Ateast one angle is >90 degrees.

Triangle Inequality

- The triangle inequality states that the sum of the lengths of any two sides of a triangle is always greater than that of the third side.
- If a, b and c are the lengths of the three sides of the triangle, then we have:

$$a + b > c$$
$$b + c > a$$
$$c + a > b$$

For example, you can never have a triangle whose sides are 5 cm, 5 cm and 10 cm as $5 + 5 = 10$, not greater than the third side.

Corollary

- The difference in lengths of any two sides is always lesser than the length of the third side.

$$a - b < c$$
$$b - c < a$$
$$c - a < b$$

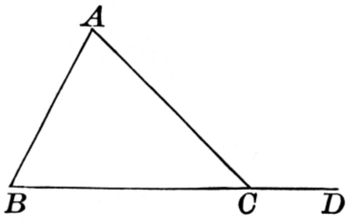
- In a triangle, the sides opposite larger angles are longer in length.

Angle Sum Property of a Triangle

The sum of the angles at the vertices of a triangle is always equal to a straight angle, i.e. $180°$. If A, B and C are the angles at the vertices of the triangle ABC, then we have $A + B + C = 180°$.

Exterior Angle Property of a Triangle

An exterior angle of triangle is the angle formed between one side (AC) of the triangle and an adjacent side (BC) that is extended (CD), as shown in the figure below.



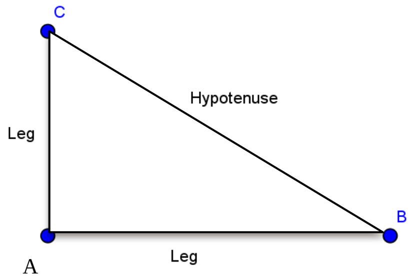
Note that the exterior angle at $C(\angle ACD)$ and the angle at the vertex $C(\angle BCA)$ of triangle ABC form a linear pair.

The exterior angle property states that the exterior angle ACD is equal to the sum of the other two interior angles at A and B.

$$\angle ACD = \angle A + \angle B$$

Pythagoras Theorem

A right-angled triangle or a right triangle is one in which one the angles at the vertices is a right angle. The side opposite the right angle is called the hypotenuse of the triangle and the sides adjacent to right triangle are called legs of the triangle.



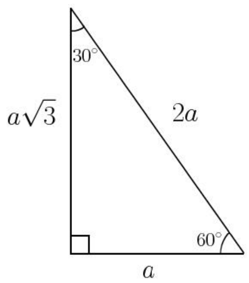
In the above triangle, BC is the hypotenuse and AB, CA are the legs of the triangle.
Pythagoras theorem states that "the sum of the squares of the legs is equal to square of the hypotenuse".

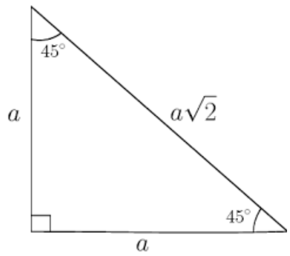
$$BC^2 + CA^2 = AB^2$$

Conversely, if the sum of the squares of two sides of a triangle is equal to the square of the third side, then the triangle is right angled with third side being the hypotenuse.

Special Right Triangles

The $30-60-90$ triangle is one in which the angles at the vertices are $30°, 60°$ and $90°$. In this triangle the sides opposite to these angles are always in the ratio $1:\sqrt{3}:2$.

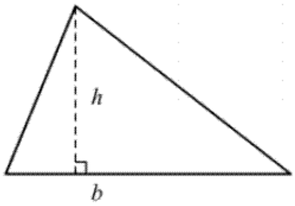




The $45-45-90$ triangle is an isosceles right triangle where two of the angles are $45°$ and the third angle is $90°$. In this triangle the sides opposite to these angles are in the ratio $1:1:\sqrt{2}$.

Area of Triangle

The area of a triangle is found by dropping a perpendicular from one vertex to the opposite side.



The length of the perpendicular from the vertex to the opposite side is called the height of the triangle (denoted by h) and the side perpendicular to the height is called the base (denoted by b) of the triangle.

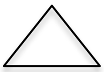
$$\text{Area}=\frac{1}{2}\times b\times h$$

For an equilateral triangle whose sides are all equal to a, the area is given by

$$\text{Area}=\frac{\sqrt{3}a^2}{4}$$

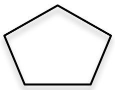
Polygons

A polygon is a planar figure that is bounded by straight edges closing in a loop to form a closed figure.

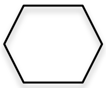


The triangle is the simplest polygon with 3 sides.

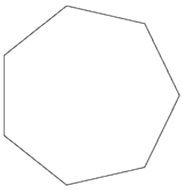
A quadrilateral has 4 sides



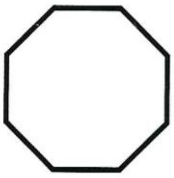
A pentagon has 5 sides.



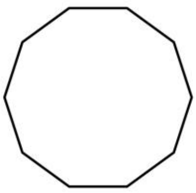
A hexagon has 6 sides



A heptagon has 7 sides



An Octagon has 8 sides



A Decagon has 10 sides.

For a polygon with n sides or n vertices, the sum of the angles at the vertices is given by $(n-2)\times 180°$.

Quadrilaterals

Quadrilaterals are polygons with 4 vertices and 4 edges. The sum of the angles of a quadrilateral is 360°. The various types of quadrilaterals and their properties are discussed below.

Trapezoid

A trapezoid is a quadrilateral in which two of the sides are parallel.

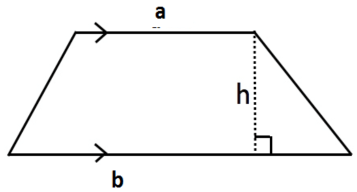
- 

Fig.6

Area of a trapezoid is given by $\frac{1}{2} \times (a + b) \times h$, where a and b are the lengths of the parallel sides and h is the distance between the parallel sides.

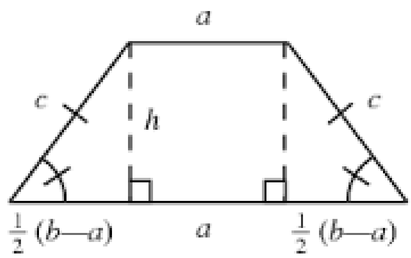


Fig.7

- An isosceles trapezoid is a special trapezoid where the non-parallel sides are equal in length.
- The diagonals of an isosceles trapezoid are equal in length.

Parallelogram

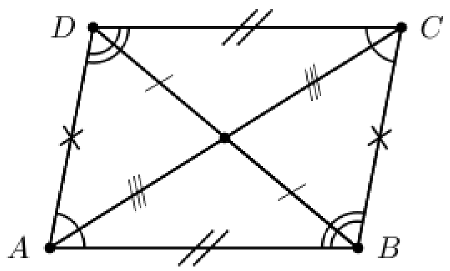
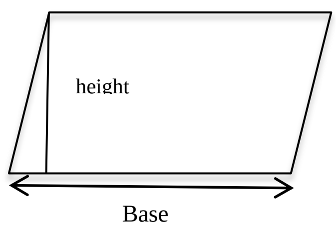


Fig.8

A parallelogram is a quadrilateral in which two pairs of opposite sides are parallel. Other properties of a parallelogram include:

- Opposite angles are equal in measure - $< DAB \leq DCB; < ABC \leq ADC$
- Adjacent angles are supplementary - $< A + < D = 180° \leq B + < C$
- Opposite sides are equal in length - $AB = CD;\ AD = BC$
- Diagonals bisect each other. – $AO = OC; DO = BO$



- Area of a parallelogram is given by $b \times h$, where b is the length of one pair of parallel sides and h is the perpendicular distance between them.

Rectangle

A rectangle is a special type of parallelogram in which all the interior angles are right angles. Other properties of a rectangle include:

- Opposite sides are equal in length
- Diagonals are equal in length
- Diagonals bisect each other.
- If l is the length of the rectangle and b is the breadth of the rectangle, then the length of the diagonal is $\sqrt{l^2 + b^2}$, from Pythagoras theorem.
- Area of a rectangle is $l \times b$ and perimeter of the rectangle is $2(l + b)$.

Rhombus

A rhombus is a special type of parallelogram in which all the sides are equal in length. Other properties of a rhombus include:

- Opposite angles are equal in measure
- Adjacent angles are supplementary
- Opposite sides are equal in length
- Diagonals are perpendicular bisectors of each other.
- Area of a rhombus is given by $a \times h$, where a is the length of the sides and h is the perpendicular distance between them.

Square

A square is a special quadrilateral that is a parallelogram, a rectangle as well as a rhombus, i.e. it's a parallelogram in which all the sides are equal and all the angles are right angles. Other properties of a square include:

- Opposite sides are equal in length
- Diagonals are equal in length
- If a is the length of the side, then the length of the diagonal is given by $a\sqrt{2}$.
- Diagonals are perpendicular bisectors of each other.
- Area of a square is a^2 and the perimeter of a square is $4a$.

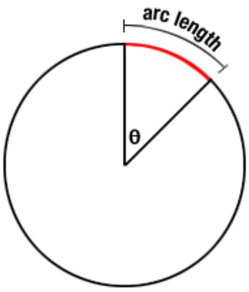
Circles

Circumference

The circumference of a circle of radius r is equal to $2\pi r$, where π is the irrational number that is defined as the ratio of circumference to diameter of any circle.

The approximate value of π is taken as $\frac{22}{7}$ or 3.14 for the sake of convenience.

Length of an Arc



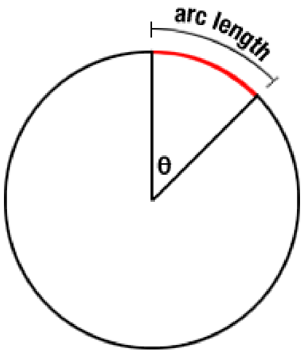
The length of an arc is dependent on the measure of the angle subtended by the arc at the center of the circle of radius r. If the measure of the angle subtended by the arc at the center is $\theta°$, then the length of the arc is given by

$$l = \frac{\theta°}{360°} \times 2\pi r$$

Area

The area of a circle with radius r is equal to πr^2

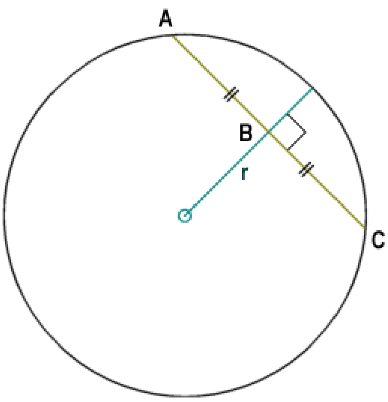
Area of a Sector



The area of a sector is dependent on the measure of the angle subtended by the arc of the sector at the center of the circle of radius r. If the measure of the angle subtended by the arc at the center is $x°$, then the area of the sector is given by

$$A = \frac{\theta°}{360°} \times \pi r^2$$

Perpendicular bisector of a Chord

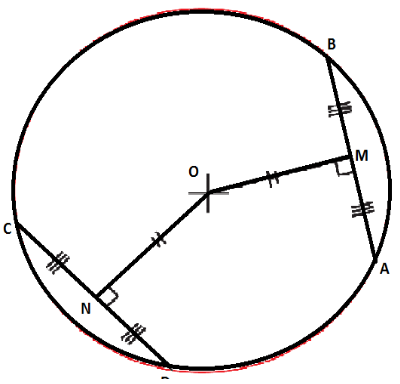


The perpendicular bisector of any chord passes through the center of the circle.

Conversely, a line drawn from the center perpendicular to a chord, bisects the chord.

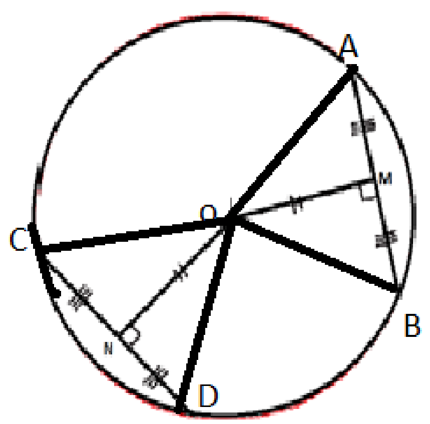
Equal Chords

In a given circle, chords that are equal in length are equidistant from the center.



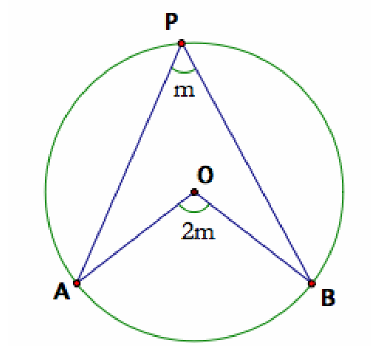
In the fig., AB and CD are equal chords. OM and ON are respectively the perpendicular bisectors of these chords. By this theorem , OM = ON.

A corollary of the above statement is that equal chords subtend equal angles at the center.



In the fig., AB and CD are equal chords and by the corollary <COD = <AOB

Central Angle Property



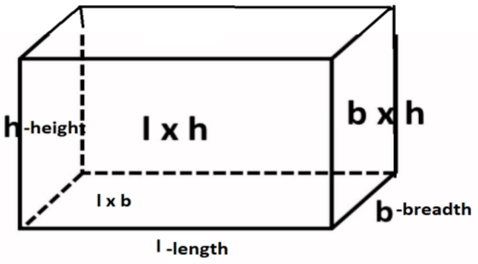
The angle subtended by an arc AB at the center O, is twice the angle subtended by it on any point, P, on the circle.

3D Figures

Cuboid

A cuboid is a three dimensional solid having 6 rectangular faces. The opposite faces of the cuboid are identical.

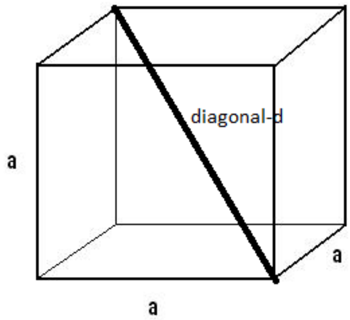
The dimensions of a cuboid are usually denoted by l (length), b (breadth) and h (height).



- The total surface area of the cuboid is given by $2(lb + bh + hl)$
- The lateral surface area (excludes the top and bottom faces) is given by $2h(l + b)$Volume of the cuboid is given by lbh.
- The length of the diagonal of a cuboid is $\sqrt{l^2 + b^2 + h^2}$

Cube

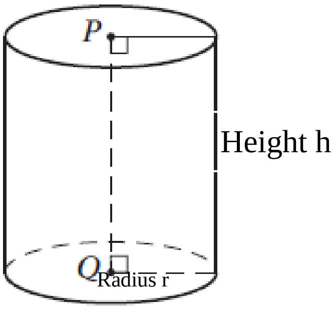
A cube is a three dimensional solid having 6 identical square faces. The length, breadth and height of a cube are equal. The length of a side of a cube is usually denoted by a.



- The total surface area of the cube is given by $6a^2$.
- The lateral surface area (excludes the top and bottom faces) is given by $4a^2$.
- Volume of the cuboid is given by a^3.
- The length of the diagonal of a cuboid is $a\sqrt{3}$.

Cylinders

A circular cylinder consists of two identical circular faces (with centers P and Q as shown in figure) and a lateral surface that consists of points that are equidistant from the line joining the centers. The line joining the centers P and Q is called the axis of the cylinder.



The radius of the circular faces is called the radius of the cylinder, r, and the distance between the centers is called the height of the cylinder, h.

- The curved surface area of the cylinder is $2\pi rh$
- The total surface area of a cylinder is $2\pi r(h + r)$
- The volume of a cylinder is $\pi r^2 h$

NOTE: The diagrams in GRE are not drawn to scale unless specified

Data Analysis

Sets

A set is well-defined collection of objects. The constituents of a set are called elements.

The **cardinality of a finite set** is defined as the number of elements in a set.

Given two sets A and B, the cardinality of the union and the intersection of two sets are related by the following equation:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

If two sets are disjoint, then $n(A \cap B) = 0$, i.e. they have no common elements

Given three sets A, B and C, the cardinality of their union is given by the following equation:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Sequences

Arithmetic Sequence

An Arithmetic Sequence or an Arithmetic Progression (A.P.) is one in which the difference between consecutive terms is a constant.

$$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$$

In general, the n^{th} term of the sequence is given by $a_n = a + (n-1)d$

The sum of the first n terms of the sequence is given by

$$S_n = \frac{n(2a + (n-1)d)}{2} = \frac{n\left(a_{\text{first\_term}} + a_{\text{last\_term}}\right)}{2}$$

Example :Consider the sequence 2,5,8,11,14,17,……

Here , the first term of the sequence is given by a .Hence the n^{th}term is given by

$$a_n = a + (n-1)d$$
$$a_n = 2 + (n-1)3 = 3n - 1$$

So we can generate the sequence using this formula.

Suppose we want to find the sum of the first 10 terms of the sequence , we directly substitute the known values in the formula , $S_{10} = \frac{10(4+(10-1)3)}{2}$=155

Geometric Sequence

A Geometric Sequence or a Geometric Progression (G.P.) is one in which the ratio between consecutive terms is a constant.

$$a, ar, ar^2, ar^3, \ldots, ar^{n-1}$$

In general, the n^{th} term of the sequence is given by $a_n = ar^{n-1}$

The sum of the first n terms of the sequence is given by $S_n = \frac{a(r^n-1)}{(r-1)}$

Example, consider the following sequence: 2,4,8,16,32,…..

Here , the common ratio is $r = \frac{4}{2} = 2$
The n^{th}term of the sequence here is $a_n = ar^{n-1}$=$2 * 2^{n-1} = 2^n$
So the sum of 10 terms(say) of this sequence will be , $S_{10} = \frac{2(2^{10}-1)}{(2-1)}$=$2(2^{10} - 1)$

Sums of a few standard sequences

- $1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$
- $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- $1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Counting Methods

Factorial

The **factorial** of a non-negative integer *n*, denoted by *n*!, is the product of all positive integers less than or equal to *n*.
Example,$5! = 5 * 4 * 3 * 2 * 1 = 120$

Some rules for factorials:

- $0! = 1$
- $n! = n(n-1)!$, for any natural number n

Permutations

- Given n distinct objects, the number of ways in which they can be arranged in a line is $n!$
- Given n distinct objects, the number of ways in which they can be arranged in a circle is $(n-1)!$
- Given n objects out which, r objects are identical and the remaining are distinct, the number of ways in which they can be arranged is $\frac{n!}{r!}$
- Given n objects, the number of ways in which r of them can be selected and then arranged is given by $^nP_r = \frac{n!}{(n-r)!}$

Example:
If there are 4 people sitting in a car , how many different arrangements of these people is possible.
Solution: Assume we have 4 slots or seats and the 4 people P1, P2,P3,P4 have to seated in them.The arrangements could be P1P2P3P4 or P1P3P2P4 or P3P1P2P4 and so on. So there are are 4 choices for the first seat. After one person is fixed in the first seat , now we need to arrange the remaining 3 persons in the remaining 3 seats .The procedure is iterative .So the total number of arrangements =$4! = 4 * 3 * 2 * 1 = 24$

Combinations

- Given n objects, the number of ways of choosing r of them is given by $^{n}C_{r} = \frac{n!}{(n-r)!\times r!}$
- Since number of ways of choosing r objects from n is same as the number of ways of leaving out $(n-r)$ objects, we have $^{n}C_{r} =^{n} C_{n-r}$
- $^{n}C_{0} =^{n} C_{n} = 1$

Example:

A basket has 5 red balls and 3 blue balls .In how many ways can we choose 2 red balls from the basket?

Solution: There are 5 red balls and we need to choose only 2 out of the 5.So the number of ways will be $5c_3$=$\frac{5!}{3!2!}$= 10

Probability

The probability of an event is defined as the ratio of number of outcomes in which the event occurs to the total number of possible outcomes of the experiment.

$$\text{probability} = \frac{\text{number of outcomes favourable to the event}}{\text{total number of possible outcomes}}$$

- Probability of an event is always in between 0 and 1, both inclusive.
- If probability of an event is 0, then we say that it is an impossible event.
- If probability of an event is 1, then we say that it is a certain event.

Examples:

Ex 1: Consider the event of tossing a coin. Find the probability of getting a head.

Solution: The sample space denotes the set of all possible outcomes . If 'H' denotes to head and 'T' denotes tail, then S = {H,T}.Let E denote the required event .E = {H}

n(S)=2 and n(E)=1.So P(E)= $\frac{n(E)}{n(S)}$=$\frac{1}{2}$

Ex2: Consider the event of rolling a die .Find the probability of getting a prime number.

Solution: The sample space S={1,2,3,4,5,6}

Let E denote the event of getting a prime number. E ={2,3,5}

So P(E) =$\frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

In general, for two events A and B that have probabilities of occurrence as $P(A)$ and $P(B)$, the probability that either one of them will occur is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- Two events are said to be independent if the outcome of one event does not affect the outcome of the other.

  Example: Consider a fair coin and a fair six-sided die. Let event A obtaining heads and event B rolling a 6.These are independent events.

- Two events are said to be mutually exclusive if the two of them cannot occur simultaneously.
  Example: Consider a sized-sided die , even numbered faces are colored red and the odd numbers faces colored green. Let A be the event of rolling a 6 and let B be the event of rolling a green face. A and B are mutually exclusive.

	A and B are independent events	A and B are mutually exclusive events
Either one of them occurring ie .,P(A or B)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	$P(A \cup B) = P(A) + P(B)$
Both occurring simultaneously i.e., P(A and B)	$P(A \cap B) = P(A)P(B)$	$P(A \cap B) = 0$

Statistics

Mean

Arithmetic mean of a data set with n data points, $\{x_1, x_2, x_3, \ldots, x_n\}$, is given by

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Example: The mean marks of 15 students in a class 12, 13, 14, 12, 13, 14, 15, 16, 17, 14, 15, 13, 15, 12,10

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\bar{x} = \frac{12+13+14+12+13+14+15+16+17+14+15+13+15+12+10}{15}$$

$=\frac{205}{15}$=13.67

Median

The median of a set of points denotes the middlemost point when the dataset is sorted in ascending or descending order

Median of dataset tells you that half of the data lies below the median and half of it lies above the median

The median of a data set with n data points depends on whether n is odd or even.

- If n is odd, then arrange the data set in ascending order as $\{x_1, x_2, x_3, \ldots, x_n\}$, and select the middle value – this value is the median.
- If n is even, then arrange the data set in ascending order as $\{x_1, x_2, x_3, \ldots, x_n\}$, then select the two middle values – the mean of these two values is the median.

$$\text{Median} = \begin{cases} \dfrac{x_{\left(\frac{n}{2}\right)} + x_{\left(\frac{n}{2}+1\right)}}{2}, & \text{if } n \text{ is even} \\ x_{\left(\frac{n+1}{2}\right)}, & \text{if } n \text{ is odd} \end{cases}$$

Example: Consider the dataset 2,3,4,2,2,3,4.
First sort the dataset: 2,2,2,3,3,4,4
Median is the middle value = 3

Mode

The mode of a data set with n data points is the most frequently occurring value.

Example: The mode of the dataset 2,3,4,2,2,3,4 is 2 as the frequency of 2 is the highest in this dataset.

Range

The range of a data set is defined as the difference between the maximum and minimum values in the data set.
Example: Consider the dataset 12, -10, -2, 13, -1, 4, 5
Range = Maximum value – minimum value = 13-(-10) = 23

Quartiles and Percentiles

Quartiles and Percentiles are two common measures of position.

The quartiles divide the data set into four groups of equal number of data points.

- The lowest quarter of the data lies below the first quartile
- The second quarter of the data lies between the first quartile and the median (2^{nd} quartile)
- The third quarter of the data lies between the median and the third quartile.
- The top quarter of the data lies above the third quartile.

The difference between the third and the first quartiles is called the Inter-Quartile Range (IQR).
Similarly, the percentiles divide the data set into 100 groups of equal number of data points.

If you are interested in where you stand compared to the rest of the herd , you need a statistics that reports "relative standing", and that statistics is called a percentile. The k^{th} percentile is a value in a dataset that splits the data into two pieces: the lower piece contains k percent of the data and the upper piece contains rest of the data , where k is any number between 0 and 100.

The median is the 50^{th} percentile .

Example: Here is a sample data from ETS that has converted the scores to percentiles .As you can see the percentiles and the raw scores are not proportional to each other .

Raw Score	Percentile
170	98
169	97
168	95
167	94
166	92
165	90
164	88
163	86

As you can observe , a Quant Score of 164 in GRE corresponds to 88 percentile , which means 88% of the students scored below 164 and only 12% of the students scored above 164. This score 164 is actually = 96.6% apx. i.e., $\frac{164}{170}$ * 100 = 96.6 approximately .So percentages are different from percentiles and are not to be confused with each other.

Note that, both quartiles and percentiles can only describe position of the data point and it is difficult to make any prediction of other data points based on just

Variance & Standard Deviation

The Standard Deviation of a data set with n data points, $\{x_1, x_2, x_3, \dots, x_n\}$, and a mean $\overline{x}$ is given by

$$SD = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}}$$

Variance of a data set is the square of the Standard Deviation

Variance $= SD^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$

Example: Consider the following datasets of the quantitative reasoning and verbal scores of 10 students in GRE in 2016-17 . Let's try to compute the mean , and the standard deviation to understand this concept better .

(Quantitative Reasoning scores)

Raw score(x)	Deviation from mean $(x_i - \overline{x})^2$
152	15.21
165	82.81
160	16.81
161	26.01
132	571.21
160	16.81
141	222.01
158	4.41
162	37.21
168	146.41
Total	1138.9

Verbal scores

Raw score(x)	Deviation from mean $(x_i - \overline{x})^2$
148	0.09
143	22.09
140	59.29
147	0.49
132	246.49
148	0.09
145	7.29
156	68.89
159	127.69
159	127.69
Total	

Based on quantitative Reasoning scores:

Mean = $\bar{x}$= 155.9;Variance = $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$ = $\frac{1138.9}{10}$ =113.89;SD = $\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$=10.67(appx)

Based on the verbal scores:

Mean = $\bar{x}$= 147.7; Variance = $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$ = $\frac{660.1}{10}$=66.01; SD =$\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$=8.12(appx)

Lets look at the following table to compare both scores:

2016-17	Mean	Standard deviation
Quant	155.9	10.67
Verbal	147.7	8.12

Inference

We can infer the following from this data:

- Since the mean of quant is greater than verbal scores , students have performed better in quant than Verbal
- Quant data has a higher SD than that of Verbal data. This means that the quant scores among the students have varied greatly compared to the verbal scores which is centered around 147.7.
- The SD (verbal) < SD(quant) .This means students have shown consistency in performance in verbal than in Quant. i.e., they are all having the same level of understanding of the verbal concepts than Quantitative Reasoning.