Name: KEY
Business Calculus
Midterm Exam 2

Instructions: Complete all problems, showing all relevant work. Reduce or simplify answers as much as possible without using a calculator. Circle or clearly mark your final answers.

1. [5 points each] Find the derivative of each function. Simplify if necessary.

a. $f(x) = 4x^{30} + 2x^9 - 8x^2 + 15$

$$f'(x) = 120x^{29} + 18x^8 - 16x$$

b. $y = e^{x^3+2x^2+x+100}$

$$y' = e^{x^3+2x^2+x+100}(3x^2+4x+1)$$

c. $g(x) = \dfrac{\ln x}{x^2}$

$$g'(x) = \frac{x^2\left(\frac{1}{x}\right) - \ln x(\underbrace{\cancel{x^2}}_{2x})}{(x^2)^2} = \frac{x - \cancel{x^2}^{\,2x}\ln x}{x^4} = \frac{x(1 - 2x\ln x)}{x^4} = \boxed{\frac{1 - 2x\ln(x)}{x^3}}$$

d. $h(x) = x^3 e^x$

$$h'(x) = x^3e^x + 3x^2e^x = x^2e^x(x+3)$$

e. $y = \ln(2x^2 + 4x)$

$$y' = \frac{4x+4}{2x^2+4x} = \boxed{\frac{2x+2}{x^2+2x}}$$

1 (cont'd). Find the derivative of each function. Simplify if necessary.

f. $f(x) = \log_{17}(7x)$

$$f'(x) = \frac{7}{7x\ln(17)} = \boxed{\frac{1}{x\ln(17)}}$$

g. $y = \sqrt{x^2 - 9}$

$$y' = \frac{2x}{2\sqrt{x^2-9}} = \boxed{\frac{x}{\sqrt{x^2-9}}}$$

h. $f(x) = \dfrac{x^2 + x + 1}{x^3}$ $= x^{-1} + x^{-2} + x^{-3}$

$$f'(x) = -x^{-2} - 2x^{-3} - 3x^{-4}$$

2. [10 points] Find an equation for the tangent line to the graph of $x^2 + y^2 = 25$ at the point $(3, 4)$.

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

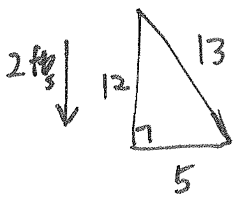
$$\frac{dy}{dx} = \frac{-x}{y} \text{ at } (3,4) = \frac{-3}{4} = m$$

$$(y - y_1) = m(x - x_1)$$

$$y - 4 = \frac{-3}{4}(x - 3)$$

$$y = \frac{-3}{4}x + \frac{9}{4} + 4 \quad \text{so} \quad \boxed{y = \frac{-3}{4}x + \frac{25}{4}}$$

3. [10 points] A 13 foot long ladder is placed against a wall. If the top of the ladder is sliding down the wall at a rate of 2 ft/s, how fast is the bottom of the ladder sliding away from the wall when the bottom is 5 feet from the base of the wall?



$$x^2 + y^2 = z^2$$

$$\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$$

$z = 13$, $x = 5$ } $y = 12$

$\frac{dy}{dt} = -2$

$\frac{dz}{dt} = 0$

$\frac{dx}{dt} = ?$

$$5\frac{dx}{dt} + 12(-2) = 0$$

$$\frac{dx}{dt} = \frac{24}{5} = \boxed{4.8 \text{ ft/s}}$$

4. [5 points each] Evaluate the limits. Show your work.

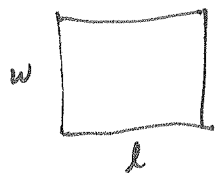
a. $\lim\limits_{x\to\infty} \frac{\ln x}{x^3} = \frac{\infty}{\infty}$ L'H

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x\to\infty} \frac{1}{3x^3} = \frac{1}{\infty} = \boxed{0}$$

b. $\lim\limits_{h\to 0} \frac{e^h - 1}{h} = \frac{e^0 - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$ L'H

$$= \lim_{h\to 0} \frac{e^h}{1} = \frac{e^0}{1} = \frac{1}{1} = \boxed{1}$$

5. [10 points] Find the dimensions of a rectangle with an area of 216 ft^2 that has the minimum perimeter. [Area $= l \cdot w$, Perimeter $= 2l + 2w$]



$A = lw = 216 \Rightarrow l = \frac{216}{w}$ or $w = \frac{216}{l}$

$P = 2l + 2w = 2l + 2\left(\frac{216}{l}\right)$

$P' = 2 - \frac{432}{l^2} = 0$

$l^2 = \frac{432}{2} = 216$

$$\boxed{\begin{array}{l} l = \sqrt{216} = 6\sqrt{6} \\ w = \sqrt{216} = 6\sqrt{6} \end{array}}$$

6. [20 points] Let $f(x) = \dfrac{3x}{x+2}$. Find all of the following.

a. What is the domain of f?

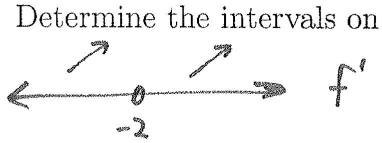
$\boxed{D_f : \{x \mid x \neq -2\}}$

b. Find $f'(x)$. Find all critical values of f.

$f'(x) = \dfrac{(x+2)(3) - (3x)}{(x+2)^2} = \boxed{\dfrac{6}{(x+2)^2}}$

CVs : top : none bott : $\boxed{x = -2}$

c. Determine the intervals on which f is increasing and/or decreasing.



$f' > 0$ always.

$\boxed{\begin{array}{l} \text{inc: } (-\infty, -2) \cup (-2, \infty) \\ \text{dec: never} \end{array}}$

d. Find all local max and min of f, if any exist.

$\boxed{\text{None}}$

Recall: $f(x) = \dfrac{3x}{x+2}$

e. Find $f''(x)$. Find all critical values of f'.

$$f''(x) = \left(6(x+2)^{-2}\right)' = -12\,(x+2)^{-3}(1) = \boxed{\dfrac{-12}{(x+2)^3}}$$

CVs: top: none
bott: $x=-2$

f. Determine the intervals on which f is concave up and/or concave down.

$+$ -2 $-$ f''

$f''(-3) = \frac{-}{-} = +$

$f(0) = \frac{-}{+} = -$

$\boxed{\text{Conc up: } (-\infty,-2) \quad \text{Conc down: } (-2,\infty)}$

g. Find all inflection points of f, if any exist.

$\boxed{\text{None}}$ b/c -2 is not in D_f!

h. Find all horizontal and vertical asymptotes of f, if any exist.

VA: $\boxed{x=-2}$

HA: $\displaystyle\lim_{x\to\pm\infty} \frac{3x}{x+2} = 3$ so $\boxed{y=3}$ is HA.

i. *Sketch* the graph of f.

Need some pts:

$f(0) = \frac{0}{2} = 0 \Rightarrow (0,0)$

$f(-4) = \frac{-12}{-2} = 6$