Name: KEY

Business Calculus
Midterm Exam 1

Instructions: Complete all problems, showing all relevant work. Reduce or simplify answers as much as possible without using a calculator. Circle or clearly mark your final answers.

1. [12 points] Let $f(x) = x^2 + 6x + 1$.

a) Write f in *vertex form* by completing the square.

$b = 6$

$\frac{b}{2} = 3$

$\left(\frac{b}{2}\right)^2 = 9$

$f(x) = (x^2 + 6x + 9) - 9 + 1$

$\boxed{f(x) = (x+3)^2 - 8}$

b) What is the vertex of the parabola given by $y = f(x)$?

$\boxed{V = (-3, -8)}$

2. [12 points] Solve each of the equations for x:

a) $81 = 9^{x^2 - x}$

$\cancel{9}^2 = \cancel{9}^{x^2 - x}$

$2 = x^2 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2,\ x = -1}$

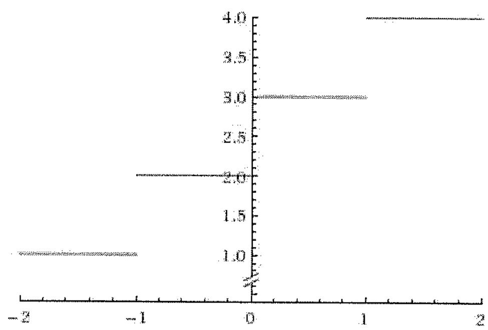
b) $\log_2(x - 2) = \cancel{32}\ 5$

$x - 2 = 2^5$

$x - 2 = 32$

$\boxed{x = 34}$

3. [12 points] Calculate the limits using the graph of $y = f(x)$.



a) $\lim_{x \to -1^-} f(x) = 1$

c) $\lim_{x \to 0} f(x) = \text{DNE}$

b) $\lim_{x \to -1^+} f(x) = 2$

d) $\lim_{x \to \frac{1}{2}} f(x) = 3$

4. [12 points] Determine where the function is continuous. Determine the type of each of its discontinuities (hole, asymptote, jump).

$$f(x) = \frac{x^2 - 5x + 6}{x^2 - x - 2} = \frac{(x-3)(x-2)}{(x-2)(x+1)}$$

$x^2 - x - 2 \neq 0$

$(x-2)(x+1) \neq 0$

$\boxed{\text{Domain: } x \neq 2, \ x \neq -1}$

Discontinuities:

$x = 2$:	hole
$x = -1$:	asymptote

$F(x) = \frac{x^2+6x+9}{x^2-9} = \frac{(x+3)(x+3)}{(x+3)(x-3)}$

5. [12 points] Let $f(x) = \cancel{\frac{x^2-9}{x^2+6x+9}} = \cancel{\frac{(x+3)(x-3)}{(x+3)(x+3)}} = \cancel{\frac{(x+3)(x-3)}{(x+3)(x+3)}}$

a) Find the domain of f.

$x^2-9 \ne 0$

$(x+3)(x-3) \ne 0$

$\boxed{x \ne \pm 3}$

b) Find $\lim_{x\to -3} f(x)$. $= \lim_{x\to -3} \frac{x+3}{x-3} = \frac{0}{-6} = \boxed{0}$

c) Find $f(-3)$. Is f continuous at $x = -3$?

$f(-3) = \frac{0}{0} = \underline{\text{undef}}$. $\underline{\text{Not cont.}}$

d) Find $\lim_{x\to\infty} f(x)$.

$\lim_{x\to\infty} \frac{x^2+6x+9}{x^2-9} = \frac{1}{1} = \boxed{1}$

6. [12 points] Let $f(x) = \begin{cases} x^2, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$.

Use the limit definition of continuous to determine if $f(x)$ is continuous at the point $x = 0$.

$$\left.\begin{aligned} \lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} x^2 = 0^2 = 0 \\ \lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} \sqrt{x} = \sqrt{0} = 0 \end{aligned}\right\} \lim_{x\to 0} f(x) = 0$$

$f(0) = \sqrt{0} = 0$

so $f(0) = \lim_{x\to 0} f(x) = 0$ and f is cont. at $x=0$.

7 - 8. Let $f(x) = x^3 + x$

7. [12 points] Find the *difference quotient* for the function $f(x)$.

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$

$$f(x+h) - f(x) = \cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x} + h - \cancel{x^3} - \cancel{x}$$

$$= 3x^2h + 3xh^2 + h^3 + h$$

$$= h\,(3x^2 + 3xh + h^2 + 1)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\cancel{h}\,(3x^2 + 3xh + h^2 + 1)}{\cancel{h}} = \boxed{3x^2 + 3xh + h^2 + 1}$$

8. [12 points] Use your answer to problem 7 and the *limit definition of derivative* to find the following:

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \left(3x^2 + 3xh + h^2 + 1\right) = \boxed{3x^2 + 1}$

b) $f'(3) = 3(3)^2 + 1 = 27 + 1 = \boxed{28}$

c) An equation for the tangent line to $y = f(x)$ at $x = 3$.

$f(3) = 3^3 + 3 = 27 + 3 = 30$

$$y - y_1 = m\,(x - x_1)$$

$$y - 30 = 28\,(x - 3)$$

$$y = 28x - 84 + 30$$

$$\boxed{y = 28x - 54}$$