8.5 Introduction to Quadratic Functions

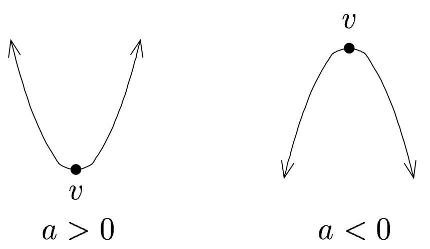
A. Definition

f is a **quadratic function** if $f(x) = ax^2 + bx + c$, where $a \neq 0$.

The graph is a **parabola**.

B. Parabolas

1. **Parabolas**



2. v is the **vertex** of the parabola

C. Formula for the Vertex

1. **Vertex Formula:** $\boxed{\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$

2. See **MTH103: College Algebra** for the justification of this formula.

D. Graphing Quadratic Functions (Parabolas)

1. Locate the vertex: use the vertex formula

2. Find the x and y intercepts.

3. Plot the points and connect in a smooth curve, recognizing whether it opens up or down.

E. Examples of Graphing

Example 1: Graph f, where $f(x) = 2x^2 + 12x + 15$

Solution

1. Locate vertex:

$$-\frac{b}{2a} = -\frac{12}{2(2)} = -\frac{12}{4} = -3$$

$$\begin{aligned} f\left(-\frac{b}{2a}\right) = f(-3) &= 2(-3)^2 + 12(-3) + 15 \\ &= 2 \cdot 9 - 36 + 15 \\ &= -18 + 15 \\ &= -3 \end{aligned}$$

Vertex Formula: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Thus the vertex is $(-3, -3)$

2. Intercepts

y-intercept:

set $x = 0$: $f(0) = 2(0)^2 + 12(0) + 15 = 15$

x-intercept:

set $y = 0$:

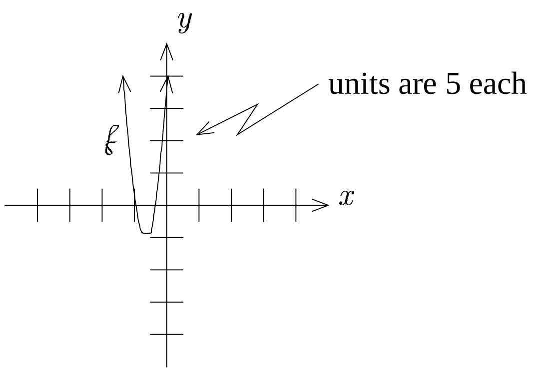
$$0 = 2x^2 + 12x + 15$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{144 - 4(2)(15)}}{2(2)}$$

$$x = \frac{-12 \pm \sqrt{144 - 120}}{4} = \frac{-12 \pm \sqrt{24}}{4} = \frac{-12 \pm 2\sqrt{6}}{4}$$

$$x = \frac{2(-6 \pm \sqrt{6})}{4} = \frac{-6 \pm \sqrt{6}}{2}$$

3. Note the graph opens upward:



Example 2: Graph f, where $f(x) = -2x^2 + 4x - 7$

Solution

1. Locate vertex:

$$-\frac{b}{2a} = -\frac{4}{2(-2)} = -\frac{4}{-4} = 1$$

$$\begin{aligned} f\left(-\frac{b}{2a}\right) = f(1) &= -2(1)^2 + 4(1) - 7 \\ &= -2 + 4 - 7 \\ &= -5 \end{aligned}$$

Vertex Formula: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

Thus the vertex is $(1, -5)$

2. Intercepts

y-intercept:

set $x = 0$: $f(0) = -2(0)^2 + 4(0) - 7 = -7$

x-intercept:

set $y = 0$:

$$0 = -2x^2 + 4x - 7$$

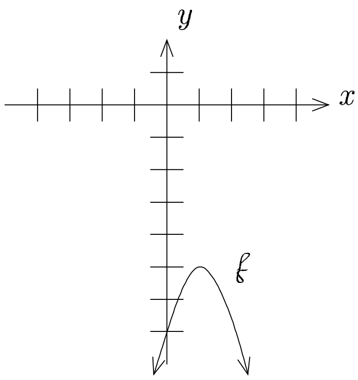
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4(-2)(-7)}}{2(-2)}$$

$$x = \frac{-4 \pm \sqrt{16 - 56}}{-4} = \frac{-4 \pm \sqrt{-40}}{-4}$$

no real solutions!

Thus, no x-intercepts.

3. Note the graph opens downward:



F. Projectile Motion

A thrown object follows a parabolic path given by

$$\boxed{\ell(t) = -16t^2 + v_0 t + h_0}$$

v_0: initial upward speed

h_0: initial height thrown from

Units of height: feet

The vertex is the peak of the path.

Features

1. The t-coordinate ("x-coordinate") of the vertex tells you **when** the projectile reaches its maximum height.

2. The "y-coordinate" of the vertex tells you what the maximum height is.

3. The projectile hits the ground when $g(t) = 0$.

G. Projectile Motion Example

A ball is thrown upward with a speed of $16\frac{\text{ft}}{\text{s}}$ from a building 32 feet high. What is the maximum height of the ball? When does it reach the ground?

Solution

Use the projectile motion model:

$$g(t) = -16t^2 + v_0 t + h_0 = -16t^2 + 16t + 32$$

Now $\quad -\dfrac{b}{2a} = -\dfrac{16}{2(-16)} = \dfrac{1}{2}$

Also,

$$\begin{aligned} g\left(-\frac{b}{2a}\right) = g\left(\frac{1}{2}\right) &= -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 32 \\ &= -16 \cdot \frac{1}{4} + 8 + 32 \\ &= -4 + 8 + 32 \\ &= 36 \end{aligned}$$

Thus the vertex is $\left(\dfrac{1}{2}, 36\right)$

Now the maximum height is the y-coordinate of the vertex: $\boxed{\text{36 feet}}$

The ball reaches the ground when $g(t) = 0$:

$$-16t^2 + 16t + 32 = 0$$

$$-16(t^2 - t - 2) = 0$$

$$-16(t - 2)(t + 1) = 0$$

$$t = 2 \quad \text{or} \quad t = -1$$

Since $t = -1$ is not physical, we must have $t = 2$.

$\boxed{\text{Thus the ball reaches the ground 2 seconds after being thrown.}}$