8.4 More on Literal Equations; Pythagorean Theorem

A. Literal Equations

When solving literal equations for a variable, sometimes roots and/or quadratic formula must be used.

Example 1: Solve $D = \dfrac{5xyw^2}{6z}$ for w

Solution

Multiply by $6z$: $6zD = 5xyw^2$

Divide by $5xy$: $\dfrac{6zD}{5xy} = w^2$

Ans $\boxed{w = \pm\sqrt{\dfrac{6zD}{5xy}}}$

Example 2: Solve $3wx^2 - 5x + w^2 = 0$ for x

Solution

Quadratic formula:

$$a = 3w, \quad b = -5, \quad c = w^2$$

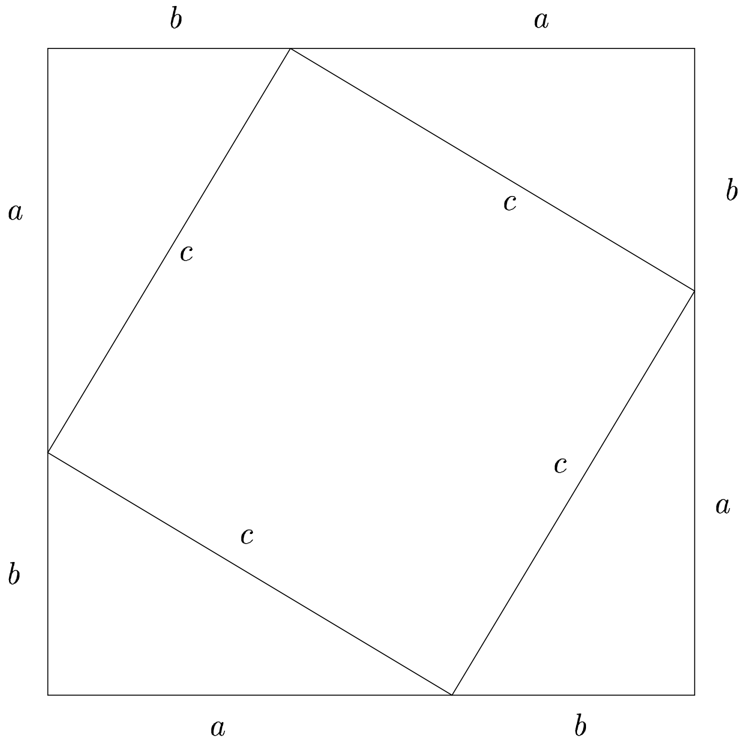
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4(3w)(w^2)}}{2(3w)}$$

Ans $\boxed{x = \dfrac{5 \pm \sqrt{25 - 12w^3}}{6w}}$

B. Pythagorean Theorem-Development

Consider the following picture:



Area of the inner square: c^2

Area of the each triangle: $\frac{1}{2}ab$

Area of the outer square: $(a+b)^2$

We see that:

$$\text{Area of the outer square} = \text{Area of the inner square} + \text{Area of the four triangles}$$

Thus $(a+b)^2 = c^2 + 4\left(\frac{1}{2}ab\right)$

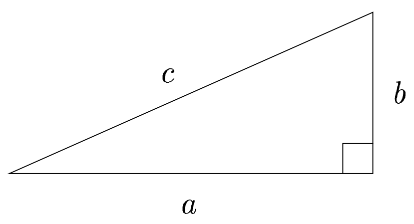
Then $a^2 + 2ab + b^2 = c^2 + 2ab$

Hence, $a^2 + b^2 = c^2$

This result is called the **Pythagorean Theorem**.

C. Pythagorean Theorem

Given a right triangle:



The sides satisfy the **Pythagorean Theorem**: $\boxed{a^2 + b^2 = c^2}$

Notation:

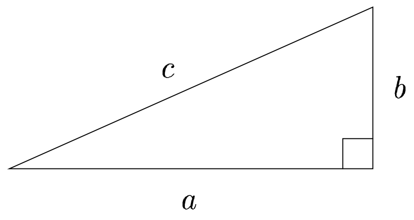
a and b are called the **legs** of the right triangle.

c is called the **hypotenuse** of the right triangle.

Note: For the legs, it doesn't matter which side you call a and which you call b.

D. Examples

Example 1: Given the right triangle:



If $a = 6$ and $b = 8$, what is c? (picture above not drawn to scale)

Solution

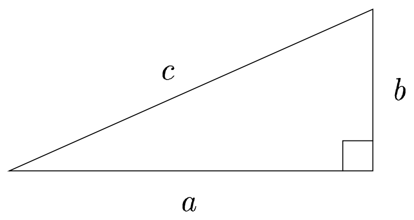
Use the Pythagorean Theorem: $a^2 + b^2 = c^2$

$$6^2 + 8^2 = c^2 \implies c^2 = 36 + 64 = 100 \implies c = \pm 10$$

Since we have a physical length, c can not be negative.

Ans $\boxed{c = 10}$

Example 2: Given the right triangle:



If $a = \sqrt{6}$ and $c = 5$, what is b? (picture above not drawn to scale)

Solution

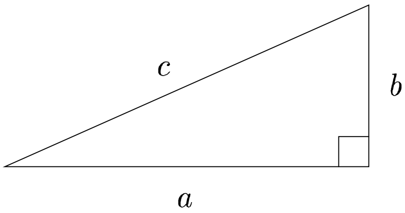
Use the Pythagorean Theorem: $a^2 + b^2 = c^2$

$$(\sqrt{6})^2 + b^2 = 5^2 \implies 6 + b^2 = 25 \implies b^2 = 19 \implies b = \pm\sqrt{19}$$

Since we have a physical length, b can not be negative.

Ans $\boxed{b = \sqrt{19}}$

Example 3: Given the right triangle:



If $c = \sqrt{15}$ and $a = 3b$, what are a and b? (picture above not drawn to scale)

Solution

Use the Pythagorean Theorem: $a^2 + b^2 = c^2$

$$(3b)^2 + b^2 = (\sqrt{15})^2 \implies 9b^2 + b^2 = 15 \implies 10b^2 = 15$$

Thus:

$$b^2 = \frac{15}{10} = \frac{3}{2} \implies b = \pm\sqrt{\frac{3}{2}}$$

Since we have a physical length, b can not be negative.

Then:

$$b = \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

Now $a = 3b$, so we have:

$$a = 3\left(\frac{\sqrt{6}}{2}\right) = \frac{3\sqrt{6}}{2}$$

Ans $\boxed{a = \frac{3\sqrt{6}}{2} \quad \text{and} \quad b = \frac{\sqrt{6}}{2}}$