8.2B Consequences of the Quadratic Formula

A. Discriminant

Since $ax^2 + bx + c = 0 \implies \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we see that if

1. $b^2 - 4ac > 0$, we get 2 real solutions
2. $b^2 - 4ac = 0$, we get 1 real solution
3. $b^2 - 4ac < 0$, we get no real solutions

$\boxed{D = b^2 - 4ac}$ is called the **discriminant**.

B. Discriminant Examples

Example 1: How many real solutions does $3x^2 - 4x + 5 = 0$ have?

Solution

$$D = b^2 - 4ac = (-4)^2 - 4(3)(5) = 16 - 60 = -44 < 0$$

Ans $\boxed{\text{no real solutions}}$

Example 2: How many real solutions does $4x^2 + 49 = 44x + 12$ have?

Solution

Move to one side: $4x^2 - 44x + 37 = 0$

$D = b^2 - 4ac = (-44)^2 - 4(4)(37) = 1344 > 0$

Ans $\boxed{\text{2 real solutions}}$

C. Writing a Quadratic Equation

If $x = c$ and $x = d$ are two solutions, then $x - c = 0$ and $x - d = 0$.
Thus $(x - c)(x - d) = 0$
Then $a(x - c)(x - d) = 0$ for a number a (usually chosen to clear fractions).

This is a quadratic equation.

Result:

Given two solutions $x = c$ and $x = d$, the quadratic equation is $a(x-c)(x-d) = 0$.

D. Examples of Writing a Quadratic Equation

Example 1: Write a quadratic equation with integer coefficients so that it has solutions $x = 3$ and $x = -4$

Solution

$a(x - 3)(x + 4) = 0 \implies a(x^2 + x - 12) = 0$
Choose $a = 1$ to get integer coefficients

Ans $\boxed{x^2 + x - 12 = 0}$

Example 2: Write a quadratic equation with integer coefficients so that it has solutions $x = -2$ and $x = \dfrac{1}{3}$

Solution

$$a\,(x+2)\left(x - \frac{1}{3}\right) = 0 \implies a\left[x^2 - \frac{1}{3}x + 2x - \frac{2}{3}\right] = 0$$

$$a\left[x^2 - \frac{5}{3}x - \frac{2}{3}\right] = 0$$

Choose $a = 3$ to clear fractions:

$$3\left(x^2 - \frac{5}{3}x - \frac{2}{3}\right) = 0 \implies 3x^2 - 5x - 2 = 0$$

Ans $\boxed{3x^2 - 5x - 2 = 0}$