8.1B Square Root Principle

A. Motivation

Recall that, for any x (positive/negative/zero), we have

$$\boxed{\sqrt{x^2} = |x|}$$

Suppose we try to solve $x^2 = 7$ by using roots.

Then we have:

$$\begin{aligned} x^2 &= 7 \\ \sqrt{x^2} &= \sqrt{7} \\ |x| &= \sqrt{7} \end{aligned}$$

This is an absolute value equation!

We recognize from Section 2.3, that the solution is $x = \sqrt{7}$ or $x = -\sqrt{7}$.

Thus, compactly, we can say that $x^2 = 7$ solves to $x = \pm\sqrt{7}$.

In equations like this, we'd like to skip all the steps and just get the answer.

Thus, we see that the result of doing roots and dealing with absolute value equations, is to have a "$\pm$" the root on the other side.

We record this as a general principle.

B. Square Root Principle

1. **Square Root Principle**

$$\boxed{u^2 = a \text{ has two solutions: } \quad u = \pm\sqrt{a}} \qquad \text{[i.e. } u = \sqrt{a} \text{ or } u = -\sqrt{a}\text{]}$$

Recall: the "$\pm$" comes from the missing steps involving an absolute value

2. In any expression, $(\text{junk})^2 = a$, we can write junk $= \pm\sqrt{a}$.

C. Examples

Example 1: Solve $(2 - 4x)^2 = 5$ for x

Solution

Apply the square root principle:

$$2 - 4x = \pm\sqrt{5}$$

Thus we have:

$$-4x = -2 \pm \sqrt{5} \implies x = \frac{-2 \pm \sqrt{5}}{-4}$$

Simplify by multiplying top and bottom by -1:

$$x = \frac{-(-2 \pm \sqrt{5})}{4} = \frac{2 \mp \sqrt{5}}{4}$$

Ans $\boxed{x = \dfrac{2 - \sqrt{5}}{4} \quad \text{or} \quad x = \dfrac{2 + \sqrt{5}}{4}}$

Example 2: Solve $(3-x)^2 = 49$ for x

Solution

Apply the square root principle:

$$3 - x = \pm\sqrt{49} = \pm 7$$

Thus we have:

$$-x = -3 \pm 7 \implies x = \frac{-3 \pm 7}{-1}$$

Multiply top and bottom by -1:

$$x = \frac{-1(-3 \pm 7)}{1} = 3 \mp 7$$

Now $x = 3 - 7$ or $x = 3 + 7$, so

Ans $\boxed{x = -4 \quad \text{or} \quad x = 10}$

D. Comments

1. The square root principle gives us another way to solve quadratic equations. Instead of moving everything to one side and factoring, we can solve the quadratic equation by writing it as

$$(\text{junk})^2 = a$$

2. **Goal:** Learn how to write a quadratic expression

$$ax^2 + bx + c \qquad \text{as} \qquad a(\text{junk})^2 + d$$

to use the square root principle on quadratic equations.