7.5 Radical Equations

A. Radical Equations

These are **equations** with radicals in them.

Here is the general strategy for solving them:

1. Isolate a radical (get one radical by itself on one side)
2. Eliminate the radical by raising each side to the appropriate power.
3. Repeat until all radicals are eliminated, and solve.
4. You **MUST** check you solutions. Some solutions are **extraneous** (i.e. fake).

B. Examples

Example 1: Solve $\sqrt[3]{2x-4}-2=0$ for x.

Solution

Isolate the radical: $\sqrt[3]{2x-4}=2$

Raise each side the third power: $(\sqrt[3]{2x-4})^3=2^3$

Thus we have:

$$2x-4=8 \implies 2x=12 \implies x=6.$$

Now we have to do a MANDATORY check:

$$\sqrt[3]{2(6)-4}-2\stackrel{?}{=}0$$

$$\sqrt[3]{12-4}-2\stackrel{?}{=}0$$

$$\sqrt[3]{8}-2\stackrel{?}{=}0$$

$$2-2\stackrel{?}{=}0\ \checkmark$$

Ans $\boxed{x=6}$

Example 2: Solve $5+\sqrt{2x+5}=x$ for x.

Solution

We **must** isolate first: $\sqrt{2x+5}=x-5$

Now square each side: $(\sqrt{2x+5})^2=(x-5)^2$

Apply the square formula on the right: $2x+5=x^2-10x+25$

We now have a quadratic equation, so we move everything to one side:

$$x^2-12x+20=0$$

We now factor this, using AntiFOIL:

$$\begin{array}{c|ll}
x^2-12x+20=0 & \boxed{20} & \text{TSP: } -,- \\
\hline
x^2-x-11x+6=0 & 11 & \\
x^2-2x-10x+6=0 & 20\ \checkmark & \\
x(x-2)-10(x-2)=0 & & \\
(x-2)(x-10)=0 & &
\end{array}$$

Now use the Zero Product Principle:

$x - 2 = 0$ OR $x - 10 = 0$

$x = 2$ OR $x = 10$

Now we have to do a MANDATORY check:

Check $x = 2$:

$$5 + \sqrt{2(2) + 5} \stackrel{?}{=} 2$$

$$5 + \sqrt{4 + 5} \stackrel{?}{=} 2$$

$$5 + \sqrt{9} \stackrel{?}{=} 2$$

$$5 + 3 \stackrel{?}{=} 2 \quad \text{X}$$

Check $x = 10$:

$$5 + \sqrt{2(10) + 5} \stackrel{?}{=} 10$$

$$5 + \sqrt{20 + 5} \stackrel{?}{=} 10$$

$$5 + \sqrt{25} \stackrel{?}{=} 10$$

$$5 + 5 \stackrel{?}{=} 10 \ \checkmark$$

Thus we have

Ans $\boxed{x = 10}$

Example 3: Solve $\sqrt{2x+3} - \sqrt{x+2} = 2$ for x.

Solution

We **must** isolate one radical first: $\sqrt{2x+3} = 2 + \sqrt{x+2}$

Now square each side: $(\sqrt{2x+3})^2 = (2 + \sqrt{x+2})^2$

Apply the square formula on the right: $2x + 3 = 4 + 4\sqrt{x+2} + (x+2)$

Now we need to isolate the remaining radical:

$2x + 3 = 6 + x + 4\sqrt{x+2}$

$4\sqrt{x+2} = x - 3$

$\sqrt{x+2} = \frac{x-3}{4}$

Now square each side again:

$(\sqrt{x+2})^2 = \left(\frac{x-3}{4}\right)^2$

$x + 2 = \frac{(x-3)^2}{16}$

Apply the square formula on the right: $x + 2 = \frac{x^2-6x+9}{16}$

Clear fractions: $x^2 - 6x + 9 = 16(x+2)$

We now have a quadratic equation, so we move everything to one side:

$x^2 - 6x + 9 = 16x + 32$

$x^2 - 22x - 23 = 0$

We now factor this, using AntiFOIL:

$$
\begin{array}{c|ll}
x^2 - 22x - 23 = 0 & \boxed{-23} & \text{TSP: } +, - \\
\hline
x^2 + x - 23x - 23 = 0 & -23\ \surd & \\
x(x+1) - 23(x+1) = 0 & & \\
(x+1)(x-23) = 0 & &
\end{array}
$$

Now use the Zero Product Principle:

$x + 1 = 0$ OR $x - 23 = 0$

$x = -1$ OR $x = 23$

Now we have to do a MANDATORY check:

Check $x = -1$:

$$\sqrt{2(-1)+3} - \sqrt{-1+2} \stackrel{?}{=} 2$$

$$\sqrt{-2+3} - \sqrt{1} \stackrel{?}{=} 2$$

$$\sqrt{1} - \sqrt{1} \stackrel{?}{=} 2$$

$$1 - 1 \stackrel{?}{=} 2 \quad \text{X}$$

Check $x = 23$:

$$\sqrt{2(23)+3} - \sqrt{23+2} \stackrel{?}{=} 2$$

$$\sqrt{49} - \sqrt{25} \stackrel{?}{=} 2$$

$$7 - 5 \stackrel{?}{=} 2\ \surd$$

Thus we have

Ans $\boxed{x = 23}$

Example 4: Solve $\sqrt{3x+4}+\sqrt{x+5}=\sqrt{7-2x}$ for x.

Solution

Isolate a radical first: in fact, the radical on the right already is!

Now square each side: $(\sqrt{3x+4}+\sqrt{x+5})^2=(\sqrt{7-2x})^2$

Apply the square formula on the left:

$$(3x+4)+2\sqrt{3x+4}\sqrt{x+5}+(x+5)=7-2x$$

Now we need to isolate another radical:

$$2\sqrt{3x+4}\sqrt{x+5}+4x+9=7-2x$$

$$2\sqrt{3x+4}\sqrt{x+5}=-2-6x$$

$$\sqrt{x+5}=\frac{-2-6x}{2\sqrt{3x+4}}$$

$$\sqrt{x+5}=\frac{-2(1+3x)}{2\sqrt{3x+4}}$$

$$\sqrt{x+5}=\frac{-1(1+3x)}{\sqrt{3x+4}}$$

Now square each side:

$$(\sqrt{x+5})^2=\left(\frac{-1(1+3x)}{\sqrt{3x+4}}\right)^2$$

$$x+5=\frac{(1+3x)^2}{(\sqrt{3x+4})^2}$$

Now apply the square formula:

$$x+5=\frac{1+6x+9x^2}{3x+4}$$

Clearing fractions: $1 + 6x + 9x^2 = (x+5)(3x+4)$

We now have a quadratic equation; multiply out and move to one side:

$$9x^2 + 6x + 1 = 3x^2 + 19x + 20$$

$$6x^2 - 13x - 19 = 0$$

We now factor this, using AntiFOIL:

$6x^2 - 13x - 19 = 0$	$\boxed{-114}$	TSP: $+, -$
$6x^2 + x - 14x - 19 = 0$	-14	
$6x^2 + 2x - 15x - 19 = 0$	-30	
jump ahead		
$6x^2 + 6x - 19x - 19 = 0$	-23 $\surd$	
$6x(x+1) - 19(x+1) = 0$		
$(x+1)(6x-19) = 0$		

Now use the Zero Product Principle:

$x + 1 = 0$ OR $6x - 19 = 0$

$x = -1$ OR $x = \frac{19}{6}$

Now we have to do a MANDATORY check:

Check $x = -1$:

$$\sqrt{3(-1)+4} - \sqrt{-1+5} \stackrel{?}{=} \sqrt{7 - 2(-1)}$$

$$\sqrt{1} + \sqrt{4} \stackrel{?}{=} \sqrt{9}$$

$$1 + 2 \stackrel{?}{=} 3 \ \surd$$

Check $x = \frac{19}{6}$:

$$\sqrt{3(\tfrac{19}{6}) + 4} + \sqrt{\tfrac{19}{6} + 5} \stackrel{?}{=} \sqrt{7 - 2(\tfrac{19}{6})}$$

$$\sqrt{\tfrac{19}{2} + \tfrac{4}{1}} + \sqrt{\tfrac{19}{6} + \tfrac{5}{1}} \stackrel{?}{=} \sqrt{\tfrac{7}{1} - \tfrac{19}{3}}$$

$$\sqrt{\tfrac{19}{2} + \tfrac{8}{2}} + \sqrt{\tfrac{19}{6} + \tfrac{30}{6}} \stackrel{?}{=} \sqrt{\tfrac{21}{3} - \tfrac{19}{3}}$$

$$\sqrt{\tfrac{27}{2}} + \sqrt{\tfrac{49}{6}} \stackrel{?}{=} \sqrt{\tfrac{2}{3}}$$

$$\tfrac{\sqrt{27}}{\sqrt{2}} + \tfrac{\sqrt{49}}{\sqrt{6}} \stackrel{?}{=} \tfrac{\sqrt{2}}{\sqrt{3}}$$

$$\tfrac{3\sqrt{3}}{\sqrt{2}} + \tfrac{7}{\sqrt{6}} \stackrel{?}{=} \tfrac{\sqrt{2}}{\sqrt{3}}$$

$$\tfrac{3\sqrt{3}\cdot\sqrt{2}}{2} + \tfrac{7\sqrt{6}}{6} \stackrel{?}{=} \tfrac{\sqrt{2}\sqrt{3}}{3}$$

$$\tfrac{3\sqrt{6}}{2} + \tfrac{7\sqrt{6}}{6} \stackrel{?}{=} \tfrac{\sqrt{6}}{3}$$

$$\tfrac{9\sqrt{6}}{6} + \tfrac{7\sqrt{6}}{6} \stackrel{?}{=} \tfrac{2\sqrt{6}}{6}$$

$$\tfrac{16\sqrt{6}}{6} \stackrel{?}{=} \tfrac{2\sqrt{6}}{6} \quad \text{X}$$

Thus we have

Ans $\boxed{x = -1}$