7.4D Rationalization

A. Introduction

We do **not** consider fractions with roots in the denominator to be completely simplified.

For instance, an example would be $\frac{7}{\sqrt{3}}$.

To simplify these, we use a different "division-type" process called **rationalization**.

B. Rationalizing Idea

We can get rid of roots in the denominator by multiplying top and bottom of the fraction by something to get rid of the root.

C. One Term Denominators

We multiply top and bottom by the root needed to make the bottom come out "clean".

D. Examples

Example 1: Simplify $\frac{5}{2\sqrt{3}}$.

Solution

We want to multiply top and bottom by something to evaluate the square root.

Multiply top and bottom by $\sqrt{3}$ to get $\sqrt{9} = 3$:

$$\frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

Ans $\boxed{\frac{5\sqrt{3}}{6}}$

Example 2: Simplify $\frac{6x^2y}{\sqrt[3]{9x}}$.

Solution

We need another 3 and x^2 in the root in the bottom to make it come out nice:

$$\frac{6x^2y}{\sqrt[3]{9x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{6x^2y\sqrt[3]{3x^2}}{3x}$$

Thus we have

Ans $\boxed{2xy\sqrt[3]{3x^2}}$

Example 3: Simplify $\frac{\sqrt[3]{6y}+2}{5\sqrt[3]{2x^2}}$.

Solution

We need another 4 and an x in the root in the bottom to make it come out nice:

$$\frac{\sqrt[3]{6y}+2}{5\sqrt[3]{2x^2}} \cdot \frac{\sqrt[3]{4x}}{\sqrt[3]{4x}} = \frac{(\sqrt[3]{6y}+2)\sqrt[3]{4x}}{5(2x)} = \frac{\sqrt[3]{24xy}+2\sqrt[3]{4x}}{10x}$$

Simplifying the first root, we get

$$\frac{2\sqrt[3]{3xy}+2\sqrt[3]{4x}}{10x} = \frac{2(\sqrt[3]{3xy}+\sqrt[3]{4x})}{10x}$$

Ans $\boxed{\frac{\sqrt[3]{3xy}+\sqrt[3]{4x}}{5x}}$

E. Binomial Denominators – Introduction

Consider $\frac{2}{\sqrt{x}+\sqrt{y}}$:

What do we multiply top and bottom by to get rid of **all** roots?

Note: $\sqrt{x}+\sqrt{y}$ does not work as a choice since:

$$(\sqrt{x}+\sqrt{y})(\sqrt{x}+\sqrt{y}) = (\sqrt{x}+\sqrt{y})^2 = x + 2\sqrt{xy} + y$$

F. Rationalizing Binomial Denominators with Square Roots

Since by FL, $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = x - y$, the general strategy is to multiply the top and bottom by the **conjugate** of the denominator.

G. Examples

Example 1: Simplify $\frac{3}{\sqrt{6}-\sqrt{5}}$.

Solution

Multiplying top and bottom by the conjugate of $\sqrt{6}-\sqrt{5}$:

$$\frac{3}{\sqrt{6}-\sqrt{5}} \cdot \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{3(\sqrt{6}+\sqrt{5})}{6-5} = \frac{3(\sqrt{6}+\sqrt{5})}{1}$$

Ans $\boxed{3(\sqrt{6}+\sqrt{5})}$

Example 2: Simplify $\frac{\sqrt{2}-\sqrt{y}}{3x\sqrt{2}+4\sqrt{y}}$.

Solution

Multiplying top and bottom by the conjugate of $3x\sqrt{2}+4\sqrt{y}$:

$$\frac{\sqrt{2}-\sqrt{y}}{3x\sqrt{2}+4\sqrt{y}} \cdot \frac{3x\sqrt{2}-4\sqrt{y}}{3x\sqrt{2}-4\sqrt{y}}$$

Using FOIL on top, and FL on the bottom, we get:

$$\frac{3x\cdot 2-4\sqrt{2y}-3x\sqrt{2y}+4y}{9x^2\cdot 2-16\cdot y}$$

Simplifying and collecting like terms, we get:

$$\frac{6x-(4+3x)\sqrt{2y}+4y}{18x^2-16y}$$

Ans $\boxed{\frac{6x-(3x+4)\sqrt{2y}+4y}{2(9x^2-8y)}}$

H. Rationalizing Binomials with Cube Roots and Higher

Suppose we have $\frac{5}{\sqrt[3]{7}-\sqrt[3]{6}}$.

The conjugate does not work here!:

$(\sqrt[3]{7}-\sqrt[3]{6})(\sqrt[3]{7}+\sqrt[3]{6}) = \sqrt[3]{49}-\sqrt[3]{36}$, which fails to get rid of the roots.

Correct choice: $\sqrt[3]{49}+\sqrt[3]{42}+\sqrt[3]{36}$!

Why? **Hint:** Difference of Cubes Formula