7.4B   Square Formula and Powers with Radicals

A. Square Formula

Recall the square formula:

$$\textbf{Square formula}: \begin{cases} 1.\text{Square the first term} \\ 2.\text{Product times two} \\ 3.\text{Square the last term} \end{cases}$$

After applying the square formula with radicals, remember to simplify.

B. Examples

Example 1:   Simplify $(\sqrt{6} - \sqrt{3})^2$.

Solution

Apply the square formula:

$$6 - 2\sqrt{6}\sqrt{3} + 3$$

$$6 - 2\sqrt{18} + 3$$

$$9 - 2\sqrt{18}$$

$$9 - 2\sqrt{9 \cdot 2}$$

Thus we get

Ans $\boxed{9 - 6\sqrt{2}}$

Example 2: Simplify $(\sqrt{2x + 3} + 5)^2$.

Solution

By the square formula, we have

$$(2x + 3) + 10\sqrt{2x + 3} + 25$$

Combining like terms, we have

Ans $\boxed{2x + 28 + 10\sqrt{2x + 3}}$

Example 3: Simplify $(\sqrt{x + 4} - \sqrt{1 - 2x})^2$.

Solution

By the square formula, we have

$$(x + 4) - 2\sqrt{x + 4}\sqrt{1 - 2x} + (1 - 2x)$$

Combining like terms, we have

Ans $\boxed{5 - x - 2\sqrt{x + 4}\sqrt{1 - 2x}}$ OR $\boxed{5 - x - 2\sqrt{(x + 4)(1 - 2x)}}$

Example 4: Simplify $(\sqrt[3]{x-1} + \sqrt{x-1})^2$.

Solution

By the square formula, we have

$$(\sqrt[3]{x-1})^2 + 2\sqrt[3]{x-1}\sqrt{x-1} + (x-1)$$

Bringing the power to the inside and rearranging, we have

Ans $\boxed{\sqrt[3]{(x-1)^2} + 2\sqrt{x-1}\sqrt[3]{x-1} + x - 1}$

C. Powers

Recall that we use the square formula repeatedly with FOIL/factor table.

Example 1: Simplify $(\sqrt{x+3} - \sqrt{x-3})^3$.

Solution

$$(\sqrt{x+3} - \sqrt{x-3})^3 = (\sqrt{x+3} - \sqrt{x-3})^2(\sqrt{x+3} - \sqrt{x-3})$$

Now apply the square formula:

$$\left[(x+3) - 2\sqrt{x+3}\sqrt{x-3} + (x-3)\right](\sqrt{x+3} - \sqrt{x-3})$$

Combining like terms:

$$(2x - 2\sqrt{x+3}\sqrt{x-3})(\sqrt{x+3} - \sqrt{x-3})$$

Now use FOIL to get the answer:

Ans $\boxed{2x\sqrt{x+3} - 2x\sqrt{x-3} - 2(x+3)\sqrt{x-3} - 2(x-3)\sqrt{x+3}}$

Example 2: Simplify $(\sqrt[3]{x+2} + \sqrt[3]{2x-1})^3$.

Solution

$$(\sqrt[3]{x+2} + \sqrt[3]{2x-1})^3 = (\sqrt[3]{x+2} + \sqrt[3]{2x-1})^2(\sqrt[3]{x+2} + \sqrt[3]{2x-1})$$

Now apply the square formula:

$$\left[(\sqrt[3]{x+2})^2 + 2(\sqrt[3]{x+2})(\sqrt[3]{2x-1}) + (\sqrt[3]{2x-1})^2\right](\sqrt[3]{x+2} + \sqrt[3]{2x-1})$$

Use the factor table:

	$(\sqrt[3]{x+2})^2$	$2(\sqrt[3]{x+2})(\sqrt[3]{2x-1})$	$(\sqrt[3]{2x-1})^2$
$\sqrt[3]{x+2}$	$x+2$	$2(\sqrt[3]{x+2})^2(\sqrt[3]{2x-1})$	$(\sqrt[3]{x+2})(\sqrt[3]{2x-1})^2$
$+\sqrt[3]{2x-1}$	$(\sqrt[3]{x+2})^2(\sqrt[3]{2x-1})$	$2(\sqrt[3]{x+2})(\sqrt[3]{2x-1})^2$	$2x-1$

Thus we have

Ans $\boxed{3x + 1 + 3(\sqrt[3]{x+2})^2(\sqrt[3]{2x-1}) + 3(\sqrt[3]{x+2})(\sqrt[3]{2x-1})^2}$