7.4A Multiplying Radicals

A. Radical Product Rule

We have the **radical product rule**: $\boxed{\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}}$ (true if a and b are positive)

Thus, for example, $\sqrt[3]{3}\sqrt[3]{9} = \sqrt[3]{27} = 3$.

This rule is really a property of exponents in disguise. Why?

Note: To use the radical product rule, the index must be the same for each radical.

B. Multiplying Radicals

We multiply by using the radical product rule. Remember to simplify afterward.

Example 1: Multiply $(2\sqrt{6})(4\sqrt{3})$.

Solution

Multiply integers and multiply roots: $(2 \cdot 4)(\sqrt{6} \cdot \sqrt{3}) = 8\sqrt{18}$

Simplify: $8\sqrt{18} = 8\sqrt{9 \cdot 2} = 8 \cdot 3\sqrt{2}$

Ans $\boxed{24\sqrt{2}}$

Example 2: Multiply $\sqrt[3]{xy^2}(4\sqrt[3]{x^2} - \sqrt[3]{y^2} + \sqrt{x})$.

Solution

Use the distributive property:

$$4\sqrt[3]{x^3y^2} - \sqrt[3]{xy^4} + \sqrt{x}\,\sqrt[3]{xy^2}$$

Simplify:

$$4x\sqrt[3]{y^2} - y\sqrt[3]{xy} + \sqrt{x}\,\sqrt[3]{xy^2}$$

We can't simplify anymore (the two radicals of the last term have different indices)!

Ans $\boxed{4x\sqrt[3]{y^2} - y\sqrt[3]{xy} + \sqrt{x}\,\sqrt[3]{xy^2}}$

Example 3: Multiply $(\sqrt[4]{x^3} + 3\sqrt[4]{x})(\sqrt[4]{2} - 3\sqrt[4]{x^3})$.

Solution

Use FOIL:

$$\sqrt[4]{2x^3} - 3\sqrt[4]{x^6} + 3\sqrt[4]{2x} - 9\sqrt[4]{x^4}$$

Simplify:

Ans $\boxed{\sqrt[4]{2x^3} - 3x\sqrt[4]{x^2} + 3\sqrt[4]{2x} - 9x}$

Example 4: Multiply $(\sqrt[3]{4} - 5\sqrt[3]{2x} + \sqrt{2x})(\sqrt{2x} - 2\sqrt[3]{2x^2})$.

Solution

Use the factor table:

	$\sqrt[3]{4}$	$-5\sqrt[3]{2x}$	$\sqrt{2x}$
$\sqrt{2x}$	$\sqrt{2x}\sqrt[3]{4}$	$-5\sqrt{2x}\sqrt[3]{2x}$	$(\sqrt{2x})^2$
$-2\sqrt[3]{2x^2}$	$-2\sqrt[3]{8x^2}$	$10\sqrt[3]{4x^3}$	$-2\sqrt{2x}\sqrt[3]{2x^2}$

Thus we have

$$\sqrt{2x}\sqrt[3]{4} - 5\sqrt{2x}\sqrt[3]{2x} + (\sqrt{2x})^2 - 2\sqrt[3]{8x^2} + 10\sqrt[3]{4x^3} - 2\sqrt{2x}\sqrt[3]{2x^2}$$

Simplifying, we get

Ans $\boxed{\sqrt{2x}\sqrt[3]{4} - 5\sqrt{2x}\sqrt[3]{2x} + 2x - 4\sqrt[3]{x^2} + 10x\sqrt[3]{4} - 2\sqrt{2x}\sqrt[3]{2x^2}}$