7.3B   Adding and Subtracting Radicals

A.   Adding and Subtracting Like Terms

The object underneath the radical is called the **radicand**.

Radicals with the same index and the same radicand are considered like terms.

$2\sqrt[3]{4}$ and $4\sqrt[3]{4}$ are like terms

$2\sqrt{6}$ and $5\sqrt[3]{6}$ are not like terms (index is different)

$2\sqrt{5}$ and $2\sqrt{6}$ are not like terms (radicand is different)

We can add or subtract like terms.

B.   Examples

Example 1:   Combine $3\sqrt[3]{4} - \sqrt[3]{4} + 4\sqrt[3]{4}$.

Solution

We just add/subtract (keeping the $\sqrt[3]{4}$ separate)

Ans $\boxed{6\sqrt[3]{4}}$

Example 2:   Combine $3\sqrt{2} - \sqrt{3x} + 5\sqrt{2} - 3\sqrt{3x}$.

Solution

Ans $\boxed{8\sqrt{2} - 4\sqrt{3x}}$

C.   Combining Radicals

To combine radicals, we may need to simplify each radical first.

Example 1:   Combine $\sqrt{54} - 4\sqrt{2} + \sqrt{8}$.

Solution

$$\sqrt{54} - 4\sqrt{2} + \sqrt{8} = \sqrt{9 \cdot 6} - 4\sqrt{2} + \sqrt{4 \cdot 2} = 3\sqrt{6} - 4\sqrt{2} + 2\sqrt{2}$$

Ans $\boxed{3\sqrt{6} - 2\sqrt{2}}$

Example 2:   Combine $5\sqrt[3]{2x^5y} + x\sqrt[3]{128x^2y}$.

Solution

$$5\sqrt[3]{2x^5y} + x\sqrt[3]{64 \cdot 2x^2y}$$

$$5\sqrt[3]{2x^5y} + 4x\sqrt[3]{2x^2y}$$

$$5x\sqrt[3]{2x^2y} + 4x\sqrt[3]{2x^2y}$$

Ans $\boxed{9x\sqrt[3]{2x^2y}}$

Example 3:   Combine $\sqrt{9x^3y^5} - 2\sqrt{24} + \sqrt{54} - x\sqrt{xy^5}$.

Solution

$$3\sqrt{x^3y^5} - 2\sqrt{4 \cdot 6} + \sqrt{9 \cdot 6} - x\sqrt{xy^5}$$

$$3\sqrt{x^3y^5} - 4\sqrt{6} + 3\sqrt{6} - x\sqrt{xy^5}$$

$$3xy^2\sqrt{xy} - 4\sqrt{6} + 3\sqrt{6} - xy^2\sqrt{xy}$$

Ans $\boxed{2xy^2\sqrt{xy} - \sqrt{6}}$