7.2A Introduction to Roots

A. Square Roots

$\sqrt{\cdot} \implies$ means find the **positive** number (or zero) that multiplied by itself gives the number inside

$\sqrt{4} = 2$, because $2 \cdot 2 = 4$

$\sqrt{81} = 9$, because $9 \cdot 9 = 81$

$\sqrt{0} = 0$, because $0 \cdot 0 = 0$

Note: Any number multiplied by itself is **never** negative! Thus, there is **no** real number answer to the square root of a negative number.

Thus, $\sqrt{-9}$ is **not** a real number.

B. Cube Roots

$\sqrt[3]{\cdot} \implies$ means find the number that multiplied by itself "3 times" gives the inside number

Thus, $\sqrt[3]{64} = 4$, because $4 \cdot 4 \cdot 4 = 64 \quad [4^3 = 64]$

Note: Cube roots can accept negative numbers, and are allowed to give back negative answers!

$\sqrt[3]{-8} = -2$, because $(-2)(-2)(-2) = -8 \quad [(-2)^3 = -8]$

C. Roots of Higher Order

We can consider roots of higher order, i.e. $\sqrt[4]{\cdot}$, $\sqrt[5]{\cdot}$, $\sqrt[6]{\cdot}$.

Note that a number inside a root indicates its type. This is called the **index**.

To evaluate these roots, we proceed by analogy, but we have one rule for roots with even index (i.e. $\sqrt{\cdot}$, $\sqrt[4]{\cdot}$, $\sqrt[6]{\cdot}$) and we have another for roots with odd index (i.e. $\sqrt[3]{\cdot}$, $\sqrt[5]{\cdot}$, $\sqrt[7]{\cdot}$).

1. **Even Index Roots**:

    a. can accept only **positive** numbers (or zero)

    so $\sqrt[6]{-64}$ is **not** a real number

    b. the answer is always **positive** (or zero)

    so $\sqrt[4]{81}$ is 3 (never -3)

2. **Odd Index Roots**:

    can accept any number, and the answer can be anything (positive, negative, or zero)

D. Domain of a Root Function

Odd roots can accept **anything**, so an odd root function has a domain of **all real numbers**.

Even roots can only accept **positive numbers or zero**, so to find the domain of an even root function, do the following:

1. Set the **inside** expression ≥ 0
2. Solve the inequality.

E. Examples on the Domain of a Root Function

Example 1: Find $\mathrm{dom}\, f$, where $f(x) = \sqrt[4]{x+1}$

Solution

Even root, so set the inside ≥ 0: $x + 1 \geq 0$

Solve the inequality: $x \geq -1$

Ans $\boxed{x \geq -1}$

Example 2: Find $\mathrm{dom}\, f$, where $f(x) = \sqrt[7]{2x-7}$

Solution

Odd root, so no restrictions . . . All real numbers!

Ans $\boxed{(-\infty, \infty)}$

Example 3: Find $\mathrm{dom}\, f$, where $f(x) = \sqrt{2-3x}$

Solution

Even root, so set the inside ≥ 0: $2 - 3x \geq 0$

Solve the inequality: $-3x \geq -2 \implies x \leq \frac{2}{3}$

Ans $\boxed{x \leq \frac{2}{3}}$

F. Graphing Root Functions

Before graphing a root function, it is useful to determine its domain first. This will guide you as to which points you should try plotting (as we will graph by point-plotting).

Example: Graph f, where $f(x) = \sqrt[4]{2x-4}$

Solution

1. First find the domain:

   Since we have an even root, we set the inside ≥ 0: $2x - 4 \geq 0$

   Now solve it: $2x - 4 \geq 0 \implies 2x \geq 4 \implies x \geq 2$

2. Now make an xy table of points, keeping in mind that we should plug in values for x of 2 or greater **only**

x	y
2	$\sqrt[4]{2 \cdot 2 - 4} = 0$
3	$\sqrt[4]{2 \cdot 3 - 4} = \sqrt[4]{2}$
4	$\sqrt[4]{2 \cdot 4 - 4} = \sqrt[4]{4}$
$\dots$	$\dots$
10	$\sqrt[4]{2 \cdot 10 - 4} = \sqrt[4]{16} = 2$

3. Now plot the points on a set of xy axes, and connect the points in a smooth curve. Bear in mind that the graph "stops" at the point $(2, 0)$.

Ans