7.1B Rational Exponents II

A. Factoring with Rational Exponents

If we see variables raised to fractional powers, the greatest common factor includes the variable(s) raised to the **smallest** fraction.

Here we revisit factoring out the GCF from Section 5.4.

Example 1: Factor the GCF out of $8x^{\frac{7}{3}} - 4x^{\frac{11}{3}} + 12x^{\frac{5}{3}}$.

Solution

Here the smallest fractional power is $\frac{5}{3}$, so we have a GCF of $4x^{\frac{5}{3}}$.

Thus we write $4x^{\frac{5}{3}}(\qquad\qquad)$, and fill in what's left.

To fill in the inside correctly, the exponents must add to what you had previously.

Thus we have $4x^{\frac{5}{3}}(2x^{\frac{2}{3}} - 4x^{\frac{6}{3}} + 3)$.

Reducing fractions, we get

Ans $\boxed{4x^{\frac{5}{3}}(2x^{\frac{2}{3}} - 4x^{2} + 3)}$

Example 2: Factor the GCF out of $10x^{\frac{3}{8}} - 15x^{\frac{1}{3}} + 20x^{\frac{5}{6}}$.

Solution

Write each fraction in terms of the LCD of the exponents, to be able to identify the smallest power.

$\ell cm(8, 3, 6)$:

$8 = 2^3$

$3 = 3$

$6 = 2 \cdot 3$

Thus $\ell cm(8, 3, 6) = 2^3 \cdot 3 = 24$.

Thus we have $10x^{\frac{9}{24}} - 15x^{\frac{8}{24}} + 20x^{\frac{20}{24}}$.

Since the smallest power is $\frac{8}{24}$, we have a GCF of $5x^{\frac{8}{24}}$.

Thus we have $5x^{\frac{8}{24}}(2x^{\frac{1}{24}} - 3 + 4x^{\frac{12}{24}})$.

Reducing fractions, we get

Ans $\boxed{5x^{\frac{1}{3}}(2x^{\frac{1}{24}} - 3 + 4x^{\frac{1}{2}})}$

Note: A problem may ask you to factor out something other than the GCF. The technique is the same. For example, if the problem asks you to factor x out of $4x^{\frac{6}{5}} - 5x^2$, then the answer would be $x(4x^{\frac{1}{5}} - 5x)$.

B. Adding/Subtracting Rational Expressions with Rational Exponents

We revisit Section 6.2.

Here, with fractional exponents, the LCD of the "large" fractions include the variable(s) to the **highest** fractional power of those present in the denominators.

Important Point: In any adding/subtracting problem with fractional exponents, we first get rid of negative exponents.

Example 1: Combine $\dfrac{4}{x^{\frac{2}{3}}y} + \dfrac{2}{x^{\frac{8}{3}}}$.

Solution

LCD= $x^{\frac{8}{3}}y$

Thus we have,

$$\frac{4}{x^{\frac{2}{3}}y} + \frac{2}{x^{\frac{8}{3}}} = \frac{4 \cdot x^{\frac{6}{3}}}{x^{\frac{8}{3}}y} + \frac{2y}{x^{\frac{8}{3}}y}$$

Thus, we have

Ans $\boxed{\dfrac{4x^2 + 2y}{x^{\frac{8}{3}}y}}$

Example 2: Combine $3x^{-\frac{2}{3}} + x^{\frac{3}{2}} - \dfrac{6}{x^{\frac{1}{4}}}$

Solution

Get rid of negative exponents:

$$\frac{3}{x^{\frac{2}{3}}} + x^{\frac{3}{2}} - \frac{6}{x^{\frac{1}{4}}}$$

Write each fractional exponent in terms of the LCD of the exponents to determine the **largest** fractional power in the denominators.

$$\frac{3}{x^{\frac{8}{12}}} + \frac{x^{\frac{18}{12}}}{1} - \frac{6}{x^{\frac{3}{12}}}$$

Here the LCD$= x^{\frac{8}{12}}$ (ignore the power of x in the numerator)

Thus,

$$\frac{3}{x^{\frac{8}{12}}} + \frac{x^{\frac{18}{12}}}{1} - \frac{6}{x^{\frac{3}{12}}}$$

$$= \frac{3}{x^{\frac{8}{12}}} + \frac{x^{\frac{18}{12}} \cdot x^{\frac{8}{12}}}{x^{\frac{8}{12}}} - \frac{6x^{\frac{5}{12}}}{x^{\frac{8}{12}}}$$

$$= \frac{3}{x^{\frac{8}{12}}} + \frac{x^{\frac{26}{12}}}{x^{\frac{8}{12}}} - \frac{6x^{\frac{5}{12}}}{x^{\frac{8}{12}}}$$

$$= \frac{3 + x^{\frac{26}{12}} - 6x^{\frac{5}{12}}}{x^{\frac{8}{12}}}$$

Thus, upon reducing fractions, we have

Ans $\boxed{\dfrac{3 + x^{\frac{13}{6}} - 6x^{\frac{5}{12}}}{x^{\frac{2}{3}}}}$

Example 3: Simplify $x^{-\frac{1}{4}}(5x^{\frac{1}{3}}+x^{-\frac{1}{6}})$

Solution

Multiply using the distributive property; use product rule:

$$5x^{(-\frac{1}{4}+\frac{1}{3})}+x^{(-\frac{1}{4}-\frac{1}{6})}$$

Do fraction arithmetic

$$5x^{(-\frac{3}{12}+\frac{4}{12})}+x^{(-\frac{3}{12}-\frac{2}{12})}$$

$$5x^{\frac{1}{12}}+x^{-\frac{5}{12}}$$

Get rid of negative exponents

$$5x^{\frac{1}{12}}+\frac{1}{x^{\frac{5}{12}}}$$

Here the LCD= $x^{\frac{5}{12}}$

Thus, we have

$$\frac{5x^{\frac{1}{12}}\cdot x^{\frac{5}{12}}}{x^{\frac{5}{12}}}+\frac{1}{x^{\frac{5}{12}}}$$

$$\frac{5x^{\frac{6}{12}}}{x^{\frac{5}{12}}}+\frac{1}{x^{\frac{5}{12}}}$$

$$\frac{5x^{\frac{6}{12}}+1}{x^{\frac{5}{12}}}$$

Reducing fractions, we have

Ans $\boxed{\frac{5x^{\frac{1}{2}}+1}{x^{\frac{5}{12}}}}$