7.1A Rational Exponents I

A. Introduction

In this section and the next, we consider variables whose **exponents** are **fractions**, i.e. $x^{\frac{3}{8}}$. While we will give meaning to these in the later sections, we will for now review some old techniques while having fractions as exponents.

B. Review of Property of Exponent Problems

We revisit the property of exponent problems from Section 1.4.

Recall: To do simplification, we do the following:

1. Use product and quotient rules to compress the expression.
2. Use power rules to "clear parentheses".
3. Get rid of negative exponents using rules.

We may need to add/subtract/multiply/divide fractions to complete our task!

C. Examples

Example 1: Simplify $(5x^{\frac{3}{4}}y^{\frac{1}{3}})^3$.

Solution

1. No product/quotient rules.

2. Clear parentheses–"power of powers": power rule (we multiply exponents)

   $$5^3x^{\frac{9}{4}}y^1$$

3. No negative exponents.

Ans $\boxed{125x^{\frac{9}{4}}y}$

Example 2: Simplify $(x^{\frac{2}{3}}y^{-\frac{7}{6}})^{\frac{1}{2}}$.

Solution

1. No product/quotient rules.

2. Power rule: multiply exponents

   $$x^{\frac{1}{3}}y^{-\frac{7}{12}}$$

3. Get rid of negative exponents.

Ans $\boxed{\dfrac{x^{\frac{1}{3}}}{y^{\frac{7}{12}}}}$

Example 3: Simplify $\left(\dfrac{6x^{\frac{1}{2}}y^{-2}}{3x^{\frac{1}{3}}y^{\frac{3}{5}}z^{-\frac{4}{3}}}\right)^{-2}$.

Solution

1. Simplify the inside by the quotient rule: subtract exponents.

$$\left(\frac{2x^{(\frac{1}{2}-\frac{1}{3})}y^{(-2-\frac{3}{5})}}{z^{-\frac{4}{3}}}\right)^{-2}$$

$$\left(\frac{2x^{(\frac{3}{6}-\frac{2}{6})}y^{(-\frac{10}{5}-\frac{3}{5})}}{z^{-\frac{4}{3}}}\right)^{-2}$$

$$\left(\frac{2x^{\frac{1}{6}}y^{-\frac{13}{5}}}{z^{-\frac{4}{3}}}\right)^{-2}$$

2. Power rule: multiply exponents

$$\frac{2^{-2}x^{-\frac{1}{3}}y^{\frac{26}{5}}}{z^{\frac{8}{3}}}$$

3. Get rid of negative exponents.

$$\frac{y^{\frac{26}{5}}}{2^2x^{\frac{1}{3}}z^{\frac{8}{3}}}$$

Thus, we have

Ans $\boxed{\dfrac{y^{\frac{26}{5}}}{4x^{\frac{1}{3}}z^{\frac{8}{3}}}}$