6.5B Applications of Rational Equations

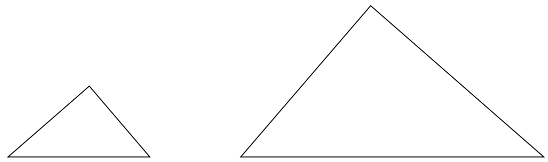
A. Proportion Problems

A **proportion** is an equation between two ratios.

Some problems explicitly mention ratios, others use the idea of **similar triangles**.

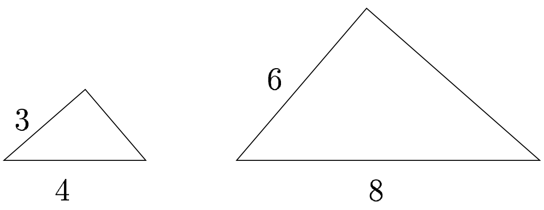
Triangles are similar if they have the same shape,
but may be of different sizes (i.e. **angles** are equal)

i.e.



are similar triangles

The sides of similar triangles have the same ratio:



Note that $\frac{3}{4} = \frac{6}{8}$.

We now look at some examples.

B. Examples

Example 1 (Problem 37): A ship in the Russian navy has a ratio of two officers for every seven seamen. The crew of the ship totals 117 people. How many officers and how many seamen are on this ship?

Solution

Let x =number of officers.
Let y =number of seamen.

Then we have two equations:

$$\begin{cases} \dfrac{x}{y} &= \dfrac{2}{7} \qquad (1) \\ x+y &= 117 \qquad (2) \end{cases}$$

From eqn (2), we have $y = 117 - x$. So substituting this into eqn (1), we get:

$$\frac{x}{117-x} = \frac{2}{7}$$

This is a rational equation!

Disallowed values: $x \neq 117$
LCD= $7(117-x)$
Multiplying both sides by the LCD:

$$7(117-x)\left[\frac{x}{117-x}\right] = 7(117-x)\left[\frac{2}{7}\right]$$

$$7x = 2(117-x)$$

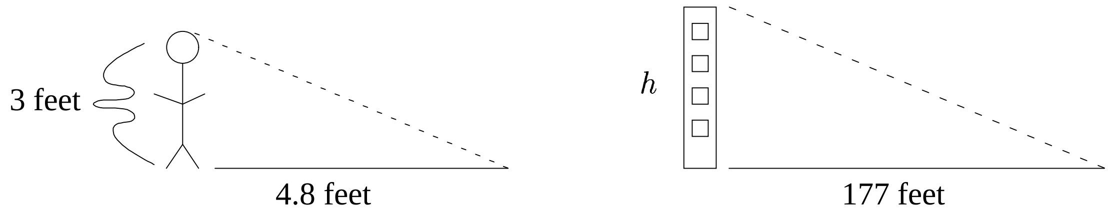
$$7x = 234 - 2x \implies 9x = 234 \implies x = 26$$

Thus $y = 117 - x = 117 - 26 = 91$.

Ans 26 officers
91 seamen

Example 2 (Problem 48): A 3-foot tall child casts a shadow that is 4.8 feet long. At the same time of day, a building casts a shadow that extends 177 feet. How tall is the building?

Solution



We have similar triangles:

$$\frac{3}{4.8} = \frac{h}{177} \implies h = 177\left(\frac{3}{4.8}\right) = 110.625$$

Ans The building is 110.625 ft tall.

C. Work Problems

People working together to complete a task often occurs as a work problem. They are solved by using the following idea:

Part of Job Done By First Person	+	Part of Job Done By Second Person	=	1	(one job done)

Now let's look at some examples.

Example 1: One person can shovel a driveway in 3 hours. For another person, it takes 6 hours. If they work together, how long will it take to shovel the driveway?

Solution

1. Person 1: Shovels driveway in 3 hours, so hourly rate is $\frac{1}{3}$ of driveway/hour.

   Person 2: Shovels driveway in 6 hours, so hourly rate is $\frac{1}{6}$ of driveway/hour.

2. Let $x =$the number of hours required to shovel a driveway

3. Then $\left(\frac{1}{3}\right)x =$amount of driveway shoveled by the first person.

   $\left(\frac{1}{6}\right)x =$amount of driveway shoveled by the second person.

4. Then $\left(\frac{1}{3}\right)x + \left(\frac{1}{6}\right)x =$amount of driveway shoveled working together

5. Want whole driveway shoveled:

   $$\frac{1}{3}x + \frac{1}{6}x = 1$$

   Multiply by LCD to clear fractions:

   $$6\left(\frac{1}{3}x + \frac{1}{6}x\right) = 6 \cdot 1$$

   Thus: $2x + x = 6 \implies 3x = 6 \implies x = 2$

Ans It will take 2 hours working together.

Example 2: Matt can paint a house in 6 hours. How long would it take for Dan to paint a house alone, if it takes 4 hours working together?

Solution

1. Matt: Paints a house in 6 hours, so hourly rate is $\frac{1}{6}$ of house per hour.

   Dan: Paints a house in x hours, so hourly rate is $\frac{1}{x}$ of house per hour.

2. Then in 4 hours, Matt paints $\left(\frac{1}{6}\right)(4)$ of house and Dan paints $\left(\frac{1}{x}\right)(4)$ of house.

3. Want whole house painted:

$$\left(\frac{1}{6}\right)(4) + \left(\frac{1}{x}\right)(4) = 1$$

$$\frac{2}{3} + \frac{4}{x} = 1$$

4. This is a rational equation:

   Disallowed values: $x \neq 0$
   LCD= $3x$
   Multiply by LCD:

$$3x\left(\frac{2}{3} + \frac{4}{x}\right) = 3x \cdot 1$$

   Thus: $2x + 12 = 3x \implies x = 12$

Ans It would take 12 hours for Dan to paint the house alone.