

6.5A Solving Literal Equations with Rational Expressions

A. Method

We combine the previous techniques of rational equations with previous literal equations. We treat all variables, other than the one we solve for, as constants.

B. Examples

Example 1: Solve $d = \frac{fl}{l+w}$ for l .

Solution

1. Denominator is factored already, and the LCD = $l + w$
2. Disallowed value for l : $l + w \neq 0 \implies l \neq -w$
3. Multiply both sides by $l + w$:

$$(l + w)d = (l + w) \left[\frac{fl}{l + w} \right]$$

$$d(l + w) = fl$$

$$dl + dw = fl$$

Now isolate l : move all terms with l to the right

$$dw = fl - dl$$

Want l by itself, so factor it out:

$$dw = l(f - d)$$

Divide both sides by $f - d$:

$$l = \frac{dw}{f - d}$$

Not disallowed!

Ans $\boxed{l = \frac{dw}{f - d}}$

Example 2: Solve $\frac{p}{3x + m} + \frac{q}{nx - 1} = 0$ for x .

Solution

1. Denominator is factored already, and the LCD = $(3x + m)(nx - 1)$

2. Disallowed values for x :

$$3x + m \neq 0 \implies x \neq -\frac{m}{3}$$

$$nx - 1 \neq 0 \implies x \neq \frac{1}{n}$$

$$(3x + m)(nx - 1) \neq 0 \implies \text{nothing new}$$

3. Multiply both sides by the LCD = $(3x + m)(nx - 1)$:

$$(3x + m)(nx - 1) \left[\frac{p}{3x + m} + \frac{q}{nx - 1} \right] = (3x + m)(nx - 1) \cdot 0$$

$$p(nx - 1) + q(3x + m) = 0$$

$$npn - p + 3qx + mq = 0$$

Now isolate x :

$$np x + 3q x = p - m q$$

Factor out x :

$$x(np + 3q) = p - m q$$

Divide both sides by $np + 3q$:

$$x = \frac{p - m q}{np + 3q}$$

Not disallowed!

Ans $\boxed{x = \frac{p - m q}{np + 3q}}$