

6.4 Rational Equations

A. Rational Equations

Equations with rational expressions are called **rational equations**

$$\text{i.e. } \frac{5}{x+1} + \frac{2}{x-3} = 6$$

Note: These problems are **not** the same type of problems as the **nonequation problems** like

$$\frac{5}{x+1} + \frac{2}{x-3}$$

Thus, our method of solution is different.

B. Method

1. Factor all denominators; determine the LCD.
2. Determine the “disallowed values”: check denominators for division by zero.
3. Multiply both sides of the equation by the LCD to clear fractions, and solve the linear equation.
4. Throw out any “fake” solutions (called **extraneous** solutions) from step 2.

C. Examples

Example 1: Solve $\frac{3x}{x-1} + 2 = \frac{3}{x-1}$ for x .

Solution

1. Denominators are factored already, and the LCD = $x - 1$

2. Disallowed value: $x - 1 \neq 0 \implies x \neq 1$

3. Multiply both sides by $x - 1$:

$$(x - 1) \left[\frac{3x}{x - 1} + 2 \right] = (x - 1) \left[\frac{3}{x - 1} \right]$$

$$3x + 2(x - 1) = 3$$

$$3x + 2x - 2 = 3$$

$$5x - 2 = 3$$

$$5x = 5$$

$$x = 1$$

4. **However**, we discard $x = 1$, so we have no solutions.

Ans $\boxed{\emptyset}$

Example 2: Solve $\frac{4}{3x-4} + \frac{2}{x+6} = \frac{5x}{3x^2+14x-24}$ for x .

Solution

1. Factor $3x^2 + 14x - 24$. Use AntiFOIL.

$3x^2 + 14x - 24$	$\boxed{-72}$	TSP: +, -
$3x^2 + 15x - 1x - 24$	-15	
$3x^2 + 16x - 2x - 24$	-32	
$3x^2 + 17x - 3x - 24$	-51	
$3x^2 + 18x - 4x - 24$	-72	✓

$$3x(x+6) - 4(x+6)$$

$$(x+6)(3x-4)$$

$$\text{Hence we have } \frac{4}{3x-4} + \frac{2}{x+6} = \frac{5x}{(x+6)(3x-4)}.$$

The LCD is $(x+6)(3x-4)$.

2. Disallowed values:

$$3x - 4 \neq 0 \implies x \neq \frac{4}{3}$$

$$x + 6 \neq 0 \implies x \neq -6$$

$$(x+6)(3x-4) \neq 0 \implies x \neq \frac{4}{3} \text{ and } x \neq -6 \text{ (already had)}$$

3. Multiply both sides by the LCD = $(x+6)(3x-4)$:

$$(x+6)(3x-4) \left[\frac{4}{3x-4} + \frac{2}{x+6} \right] = (x+6)(3x-4) \left[\frac{5x}{(x+6)(3x-4)} \right]$$

$$4(x+6) + 2(3x-4) = 5x$$

$$4x + 24 + 6x - 8 = 5x$$

$$10x + 16 = 5x$$

$$5x + 16 = 0$$

$$5x = -16$$

$$x = -\frac{16}{5}$$

4. Not a disallowed value!

Ans $\boxed{x = -\frac{16}{5}}$

Example 3: Solve $\frac{4}{x-4} - \frac{3}{x+3} = \frac{x+24}{x^2-x-12}$ for x .

Solution

1. Factor $x^2 - x - 12$. Use AntiFOIL.

$x^2 - x - 12$	$\boxed{-12}$	TSP: +, -
$x^2 + x - 2x - 12$	-2	
$x^2 + 2x - 3x - 12$	-6	
$x^2 + 3x - 4x - 12$	-12	✓

$$x(x+3) - 4(x+3)$$

$$(x+3)(x-4)$$

Hence we have $\frac{4}{x-4} - \frac{3}{x+3} = \frac{x+24}{(x-4)(x+3)}$.

The LCD is $(x-4)(x+3)$.

2. Disallowed values:

$$x - 4 \neq 0 \implies x \neq 4$$

$$x + 3 \neq 0 \implies x \neq -3$$

$$(x - 4)(x + 3) \neq 0 \implies \text{nothing new}$$

3. Multiply both sides by the LCD = $(x - 4)(x + 3)$:

$$(x - 4)(x + 3) \left[\frac{4}{x - 4} - \frac{3}{x + 3} \right] = (x - 4)(x + 3) \left[\frac{x + 24}{(x - 4)(x + 3)} \right]$$

$$4(x + 3) - 3(x - 4) = x + 24$$

$$4x + 12 - 3x + 12 = x + 24$$

$$x + 24 = x + 24$$

$$24 = 24$$

All real numbers!

4. However, we must disallow 4 and -3

Ans All real numbers except -3 and 4