6.2B Adding/Subtracting Rational Expressions

A. Method

1. Factor the denominators and find the LCD.

2. Rewrite each fraction with the common denominator by multiplying top and bottom of the original fraction by appropriate factors.

3. Add/subtract numerators to form one fraction.

4. Simplify the answer, if possible.

B. Examples

Example 1: Find $\dfrac{2x}{x^2+2x-8}+\dfrac{x-6}{x^2-3x+2}$

Solution

1. Factor the denominators:

x^2+2x-8	$\boxed{-8}$ TSP: $+,-$
$x^2+3x-x-8$	-3
$x^2+4x-2x-8$	$-8\ \surd$
$x(x+4)-2(x+4)$	
$(x+4)(x-2)$	

x^2-3x+2	$\boxed{2}$ TSP: $-,-$
$x^2-x-2x+2$	$2\ \surd$
$x(x-1)-2(x-1)$	
$(x-1)(x-2)$	

Hence we have $\dfrac{2x}{(x+4)(x-2)} + \dfrac{x-6}{(x-1)(x-2)}$

The LCD is $(x+4)(x-2)(x-1)$

2. Rewrite each fraction with LCD as new denominator:

$$\frac{2x}{(x+4)(x-2)} \cdot \frac{(x-1)}{(x-1)} + \frac{(x-6)}{(x-1)(x-2)} \cdot \frac{(x+4)}{(x+4)}$$

$$\frac{2x(x-1)}{(x+4)(x-2)(x-1)} + \frac{(x-6)(x+4)}{(x+4)(x-2)(x-1)}$$

3. Now add numerators (multiply out to combine):

$$\frac{2x^2-2x}{(x+4)(x-2)(x-1)} + \frac{x^2-2x-24}{(x+4)(x-2)(x-1)}$$

$$\frac{3x^2-4x-24}{(x+4)(x-2)(x-1)}$$

4. Now factor the top if possible to see if you can simplify:

$3x^2-4x-24$	$\boxed{-72}$ TSP: $+,-$
$3x^2+x-5x-24$	-5
$3x^2+2x-6x-24$	-12
jump ahead	
$3x^2+6x-10x-24$	-60
$3x^2+7x-11x-24$	-77

It is prime.

Ans $\boxed{\dfrac{3x^2-4x-24}{(x+4)(x-2)(x-1)}}$

Example 2: Find $\dfrac{x-1}{x-2} - \dfrac{x+1}{x+2} + \dfrac{x-6}{x^2-4}$

Solution

1. Factor the denominators:

$$\frac{x-1}{x-2} - \frac{x+1}{x+2} + \frac{x-6}{(x+2)(x-2)}$$

The LCD is $(x+2)(x-2)$.

2. Rewrite each fraction with LCD as new denominator:

$$\frac{(x+2)(x-1)}{(x+2)(x-2)} + \frac{(x+1)(x-2)}{(x+2)(x-2)} + \frac{x-6}{(x+2)(x-2)}$$

3. Expand numerators and combine:

$$\frac{x^2+x-2}{(x+2)(x-2)} - \frac{x^2-x-2}{(x+2)(x-2)} + \frac{x-6}{(x+2)(x-2)}$$

$$\frac{(x^2+x-2)-(x^2-x-2)+(x-6)}{(x+2)(x-2)} \qquad (- \text{ affects whole numerator!})$$

$$\frac{x^2+x-2-x^2+x+2+x-6}{(x+2)(x-2)}$$

$$\frac{3x-6}{(x+2)(x-2)}$$

4. Factor numerator and simplify:

$$\frac{3(x-2)}{(x+2)(x-2)} = \frac{3\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$$

Ans $\boxed{\dfrac{3}{x+2}}$

Example 3: Find $3x + 2 - \dfrac{2x+4}{x+6}$

Solution

1. Factor denominators and find the LCD:

   The denominators are already factored, and we have $\dfrac{3x+2}{1} - \dfrac{2x+4}{x+6}$

   The LCD is $x + 6$

2. Rewrite each fraction with LCD as new denominator:

$$\frac{(3x+2)(x+6)}{x+6} - \frac{2x+4}{x+6}$$

3. Subtract numerators:

$$\frac{3x^2+20x+12}{x+6} - \frac{2x+4}{x+6}$$

$$\frac{3x^2+18x+8}{x+6}$$ (remember $-$ affects whole numerator)

4. Try to factor numerator and simplify:

$3x^2+18x+8$	$\boxed{24}$ TSP: $+,+$
$3x^2+x+17x+8$	17
$3x^2+2x+15x+8$	30

   It is prime!

Ans $\boxed{\dfrac{3x^2+18x+8}{x+6}}$