6.2A Finding Common Denominators

A. Introduction

When we add/subtract fractions, we need to find a common denominator. In an algebraic fraction, we work with factors, so we need to learn how to find common denominators from factors.

B. LCM of Numbers–Motivation

If we have $\frac{5}{6} + \frac{3}{8}$, we need to find the least common denominator.
We know that the answer is 24. Let's look at how to do it from factors.

Write down the prime factorization of 6 and 8:

$$6 = 2 \cdot 3$$

$$8 = 2^3$$

Since $24 = 2^3 \cdot 3$, we see that we need to write down the product of all factors present to the highest power they appear.

C. LCM of Algebraic Expressions

To find the LCM of algebraic expressions, we use the same strategy outlined above.

1. Factor everything, if not done already

2. The LCM is the product of all the factors present to the highest (not total) power they appear.

Note: If the expressions have multiplicative numbers in front, we can find the LCM of the numbers via the old method if desired.

D. Examples

Example 1: Find $\ell cm\,((2x-3)^4(x+3)^5, (2x-3)^7(x+3))$

Solution

The highest power that we see for $(2x-3)$ is 7, and for $(x+3)$ is 5.

Ans $\boxed{(2x-3)^7(x+3)^5}$

Example 2: Find $\ell cm\,(12x^3y(z+3)^2, 18x^2y^5(z+3)^3w)$

Solution

The LCM of 12 and 18 is 36 (add 18 to itself, 36 is a multiple of 12).

Then the highest power for x is 3; for y is 5; for $(z+3)$ is 3; for w is 1

Ans $\boxed{36x^3y^5(z+3)^3w}$

Note: In the above example, you could have treated the numbers similarly. You could have written 12 as $2^2 \cdot 3$ and 18 as $2 \cdot 3^2$; then by using the "max powers" idea, you would get $2^2 \cdot 3^2 = 36$.

Example 3: Find $\ell cm\,(6xy^4(x+2)^2, 15x^2y^3(x+2)^2, 10x^5y^4z^2)$

Solution

The LCM of 6, 15, and 10 is 30.

Then the highest power for x is 5; for y is 4; for z is 2; for $(x+2)$ is 3.

Ans $\boxed{30x^5y^4z^2(x+2)^3}$

E. Finding the LCD

The LCD is simply the LCM of the denominators.

Example: Find the LCD of $\dfrac{6x}{x^2-x-12}$ and $\dfrac{x+4}{x^2-9}$

Solution

We need to find $\ell cm\,(x^2-x-12, x^2-9)$.

To do so, we need to factor each first.

Via AntiFOIL and the difference of squares, we get:

$$\ell cm\,((x+3)(x-4), (x+3)(x-3))$$

Ans $\boxed{(x+3)(x-4)(x+3)}$