6.1A Rational Expressions and Rational Functions

A. Introduction

An algebraic fraction is called a **rational expression**.

For example, $\frac{3x+4}{x-2}$ and $\frac{2x}{x^2-x-12}$ are rational expressions.

A **rational function** is a function whose output formula $f(x)$ is a rational expression.

Thus if $f(x) = \frac{2x-1}{x+3}$, then f is a rational function.

Note: Since we can't divide by zero, numbers that cause the denominator to be zero are not legitimate inputs.

Fact: The domain (set of allowed x-values) of a rational function is all real numbers **except** those that make the denominator zero.

B. Finding the Domain of a Rational Function

Set the denominator equal to zero. The "answers" are the values we have to throw away!

Example 1: Find $\mathrm{dom}\, f$ where $f(x) = \frac{3x+2}{x-5}$

Solution

Set $x - 5 = 0$

Then we have $x = 5$, which is what we need to throw away!

Ans all real numbers **except** $x = 5$

Example 2: Find $\mathcal{dom}\, f$ where $f(x) = \frac{2x-1}{5x^2-11x+2}$

Solution

Set $5x^2 - 11x + 2 = 0$

Solve this, as before: factor!

$5x^2 - 11x + 2 = 0$	$\boxed{10}$ TSP: $-, -$
$5x^2 - x - 10x + 2 = 0$	10 $\surd$
$x(5x-1) - 2(5x-1) = 0$	
$(5x-1)(x-2) = 0$	

Zero product principle:

$5x - 1 = 0$ OR $x - 2 = 0$

$x = \frac{1}{5}$ OR $x = 2$

These are the values we need to throw away!

Ans $\boxed{\text{all real numbers } \textbf{except } x = \frac{1}{5} \text{ and } x = 2}$