5.7A Generalized Factoring I

A. General Factoring Strategy

1. First try to factor out the GCF.

2. Decide how many terms you have, and do the following:

   a. **Two terms**: look for

      I. Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

      II. Difference of Cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

      III. Sum of Cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

   b. **Three terms**: look for

      I. Perfect Squares: $a^2 + 2ab + b^2 = (a+b)^2$ and $a^2 - 2ab + b^2 = (a-b)^2$

      II. AntiFOIL

   c. **Four terms or more**: try factoring by grouping; rearrange if necessary

3. After successively applying the techniques repeatedly, you reach the completely factored answer.

Remember:

   a. Some trinomials are not factorable

   b. Sum of squares $a^2 + b^2$ is not factorable

B. Examples

Example 1: Factor $3x^5 - 81x^2$ completely.

Solution

1. First factor out the GCF:

$$3x^2(x^3 - 27)$$

2. Inside: two terms; it is a difference of cubes

$$3x^2\left((x)^3 - (3)^3\right)$$

Ans $\boxed{3x^2(x-3)(x^2+3x+9)}$

Example 2: Factor $80x^3 + 36mxy^2 - 30mx^2 - 96x^2y^2$ completely.

Solution

1. First factor out the GCF:

$$2x(40x^2 + 18my^2 - 15mx - 48xy^2)$$

2. Inside: four terms; factor by grouping

$$2x[40x^2 + 18my^2 - 15mx - 48xy^2]$$

$$2x[2(20x^2 + 9my^2) - 3x(5m + 16y^2)] \quad \text{X}$$

Rearrange: Try 1 & 3 and 2 & 4

$$2x[40x^2 - 15mx + 18my^2 - 48xy^2]$$

$$2x[5x(8x - 3m) + 6y^2(3m - 8x)]$$

Negative factor pairs!

$$2x[5x(8x - 3m) - 6y^2(8x - 3m)]$$

$$2x[(8x - 3m)(5x - 6y^2)]$$

By associativity, we get

Ans $\boxed{2x(8x - 3m)(5x - 6y^2)}$

Example 3: Factor $16x^3y - 24x^2y^2 + 9xy^3$ completely.

Solution

1. First factor out the GCF:

$$xy(16x^2 - 24xy + 9y^2)$$

2. Inside: three terms; try perfect square factoring

First term: $(4x)^2$

Last term: $(3y)^2$

Test: $(4x - 3y)^2$: $(4x - 3y)^2 = 16x^2 - 24xy + 9y^2$ $\surd$

Thus we have

Ans $\boxed{xy(4x - 3y)^2}$

Example 4: Factor $12x^2y^2 + 27y^2 - 45xy^2$ completely.

Solution

1. First factor out the GCF:

   $3y^2(4x^2 + 9 - 15x)$

2. Inside: three terms

   Rearrange $3y^2(4x^2 - 15x + 9)$ and try perfect square factoring

   First term: $(2x)^2$

   Last term: $(3)^2$

   Test: $(2x-3)^2$: $(2x-3)^2 = 4x^2 - 12x + 9$ X

   This shortcut fails, so we must do AntiFOIL!

$4x^2 - 15x + 9$	$\boxed{36}$ TSP: $-, -$
$4x^2 - x - 14x + 9$	14
$4x^2 - 2x - 13x + 9$	26
$4x^2 - 3x - 12x + 9$	36 $\surd$

   $x(4x-3) - 3(4x-3)$

   $(4x-3)(x-3)$

Thus we have

Ans $\boxed{3y^2(4x-3)(x-3)}$

Example 5: Factor $128x^6 - 2$ completely.

Solution

1. First factor out the GCF:

$$2(64x^6 - 1)$$

2. Inside: two terms; it is a difference of squares

$$2\left((8x^3)^2 - (1)^2\right)$$

$$2(8x^3 + 1)(8x^3 - 1)$$

Each of these can be factored: sum and difference of cubes

$$2[(2x)^3 + 1^3][(2x)^3 - 1^3]$$

$$2[(2x+1)(4x^2 - 2x + 1)][(2x-1)(4x^2 + 2x + 1)]$$

Keeping in mind that the trinomials are prime, by associativity and commutativity, we get

Ans $\boxed{2(2x+1)(2x-1)(4x^2 - 2x + 1)(4x^2 + 2x + 1)}$