5.6 Special Factoring Formulas

A. Perfect Square Factoring

1. **Perfect Square Factoring Formulas**: $\boxed{a^2 + 2ab + b^2 = (a+b)^2}$ and $\boxed{a^2 - 2ab + b^2 = (a-b)^2}$

2. To use: if the first and last terms of a trinomial are squares, try writing a perfect square; then use the square formula to see if you are correct.

3. Examples:

Example 1: Factor $4x^2 + 12x + 9$.

Solution

Since $4x^2 = (2x)^2$ and $9 = 3^2$, we GUESS $(2x+3)^2$

Test: using the square formula, $(2x+3)^2 = 4x^2 + 12x + 9$ $\surd$

Ans $\boxed{(2x+3)^2}$

Example 2: Factor $9x^2 - 24xy + 16y^2$.

Solution

Since $9x^2 = (3x)^2$ and $16y^2 = (4y)^2$, we GUESS $(3x-4y)^2$

Test: using the square formula, $(3x-4y)^2 = 9x^2 - 24xy + 16y^2$ $\surd$

Ans $\boxed{(3x-4y)^2}$

Example 3: Factor $4x^2 - 15x + 9$.

Solution

Since $4x^2 = (2x)^2$ and $9 = 3^2$, we GUESS $(2x-3)^2$

Test: using the square formula, $(2x-3)^2 = 4x^2 - 12x + 9$ X

This shortcut fails, so we must do AntiFOIL!

$$\begin{array}{c|ll} 4x^2 - 15x + 9 & \boxed{36} & \text{TSP: } -, - \\ \hline 4x^2 - x - 14x + 9 & 14 & \\ 4x^2 - 2x - 13x + 9 & 26 & \\ 4x^2 - 3x - 12x + 9 & 36 \surd & \end{array}$$

$x(4x-3) - 3(4x-3)$

Ans $\boxed{(4x-3)(x-3)}$

B. Difference of Squares

1. **Formula**: $\boxed{a^2 - b^2 = (a+b)(a-b)}$

2. Examples:

Example 1: Factor $x^2 - 9$.

Solution

Write $x^2 - 9$ as $x^2 - 3^2$

By the formula, we get

Ans $\boxed{(x+3)(x-3)}$

Example 2: Factor $4x^2 - 49$.

Solution

Write $4x^2 - 49$ as $(2x)^2 - 7^2$

By the formula, we get

Ans $\boxed{(2x+7)(2x-7)}$

Example 3: Factor $16x^4 - 81y^4$.

Solution

Write $16x^4 - 81y^4$ as $(4x^2)^2 - (9y^2)^2$

By the formula, we get $(4x^2 + 9y^2)(4x^2 - 9y^2)$

Now $4x^2 + 9y^2$ is a sum of squares (not factorable),
but we can factor $4x^2 - 9y^2$ further as a difference of squares again!

Thus $(4x^2 + 9y^2)(4x^2 - 9y^2) = (4x^2 + 9y^2)\left((2x)^2 - (3y)^2\right)$

By the difference of squares, we get

Ans $\boxed{(4x^2 + 9y^2)(2x+3y)(2x-3y)}$

C. Difference and Sum of Cubes

1. **Formulas**

   Difference of Cubes: $\boxed{a^3 - b^3 = (a - b)(a^2 + ab + b^2)}$

   Sum of Cubes: $\boxed{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$

2. **Note**: The quadratic in the factorization is prime (no need to try to factor it!)

3. Easy way to remember these two formulas:

   First factor: just "remove" the cubes

   Second factor: pretend to "square" the first factor **EXCEPT**
   rather than doing product times 2, do product times -1

4. Examples:

   Example 1: Factor $x^3 + 27$.

   Solution

   Write $x^3 + 27$ as $x^3 + 3^3$

   By the formula, we get

   Ans $\boxed{(x + 3)(x^2 - 3x + 9)}$

Example 2: Factor $8x^3 - 125$.

Solution

Write $8x^3 - 125$ as $(2x)^3 - (5)^3$

By the formula, we get

Ans $\boxed{(2x-5)(4x^2+10x+25)}$

D. Closing Comment

As always when factoring, you should first check to see if you can factor out a GCF before trying any other technique. In the last sections, we will put all of our techniques together.