5.5B AntiFOIL

A. AntiFOIL technique

1. Given $ax^2 + bx + c$, multiply ac. This is your "magic number".

2. Split the middle term into two, with signs given by the TSP.
Start with one of the numbers as being the number 1.

3. Multiply the middle two numbers and see if they match the "magic number".

4. If they match, then factor by grouping. If not, increment the numbers by 1, and try again.

This method is easier to do than to describe.
We really just need to make a simple "table" to do this.

B. Examples

Example 1: Factor $2x^2 - 13x - 24$.

Solution

1. Magic number: $2(-24) = -48$

2. TSP: $+, -$

Thus we will split -13 into a positive and negative number. Start with 1

Make a table!

$2x^2 - 13x - 24$	$\boxed{-48}$	TSP: $+, -$
$2x^2 + x - 14x - 24$	-14	
$2x^2 + 2x - 15x - 24$	-30	
$2x^2 + 3x - 16x - 24$	-48 $\surd$	

Now factor by grouping!

$x(2x+3)-8(2x+3)$

Ans $\boxed{(2x+3)(x-8)}$

Example 2: Factor $6x^2-23x+20$.

Solution

Make a table!

$6x^2-23x+20$	$\boxed{120}$	TSP: $-,-$
$6x^2-x-22x+20$	22	
$6x^2-2x-21x+20$	42	
jump ahead (long way to go)		
$6x^2-7x-16x+20$	112	
$6x^2-8x-15x+20$	$120\ \surd$	

Now factor by grouping!

$2x(3x-4)-5(3x-4)$

Ans $\boxed{(3x-4)(2x-5)}$

Example 3: Factor $6x^2+43x-40$.

Solution

Make a table!

Note: Given the TSP, we need to put the 1 in the second spot to make positive 43.

$6x^2+43x-40$	$\boxed{-240}$	TSP: $+,-$
$6x^2+44x-x-40$	-44	
$6x^2+45x-2x-40$	-90	
$6x^2+46x-3x-40$	-138	
$6x^2+47x-4x-40$	-188	
$6x^2+48x-5x-40$	-240 $\surd$	

Now factor by grouping!

$6x(x+8)-5(x+8)$

Ans $\boxed{(x+8)(6x-5)}$

C. Prime Trinomials

A trinomial is **prime** if it can't be factored.

This happens when the magic number is never hit.

We can see this when:

1. While incrementing, you "jump over" the magic number.

2. Also possible to have the magic number never reached due to the numbers being the "wrong size"

These are really symptoms of the AntiFOIL formula never being satisfied.

D. Examples of Prime Trinomials

Example 1: Factor $4x^2 + 21x + 15$.

Solution

Make a table!

$4x^2 + 21x + 15$	$\boxed{60}$ TSP: $+, +$
$4x^2 + x + 20x + 15$	20
$4x^2 + 2x + 19x + 15$	38
$4x^2 + 3x + 18x + 15$	54
$4x^2 + 4x + 17x + 15$	68

We skipped over 60!

Ans $\boxed{4x^2 + 21x + 15 \text{ (prime)}}$

Example 2: Factor $4x^2 + 14x + 15$.

Solution

Make a table!

$4x^2 + 14x + 15$	$\boxed{60}$ TSP: $+, +$
$4x^2 + x + 13x + 15$	13
$4x^2 + 2x + 12x + 15$	24
jump ahead (long way to go)	
$4x^2 + 7x + 7x + 15$	49
$4x^2 + 8x + 6x + 15$	48

The numbers start decreasing; 60 is never reached!

Ans $\boxed{4x^2 + 14x + 15 \text{ (prime)}}$

E. Comments

There are two other extremely popular methods for factoring trinomials that the author finds inferior.

One method is called **trial and error**. This amounts to guessing the likely factors and obtaining the correct factors after eliminating wrong choices. While this method can be effective for trinomials of the form $x^2 + bx + c$ (i.e. $a = 1$) when the numbers are small, trinomials that are not extremely simple are not effectively factored by this method.

Another method is called the **grouping number method**. This method is more similar to AntiFOIL, in that the goal is factoring by grouping. However, the method for obtaining the middle terms amounts to finding all factor pairs of the magic number (in this context, the grouping number), and then trying to finding among all such factor pairs the one that adds correctly. While this method is slightly more useful than trial and error, it is horribly ineffective if the magic number/grouping number is large, because you must fill pages just to consider all of these factor pairs (consider as in Example 3 from Section B all of the factors of -240!). Furthermore, all factor pairs must be listed to establish a trinomial as prime.

The beauty of AntiFOIL is that even if the numbers are large, you should be able to factor a trinomial or determine it is prime in about 5 steps due to the organization of the table. The price, of course, is that the method takes slightly longer to explain.