

5.5A Factoring Trinomials: An Introductory Discussion

A. Introduction

Before discussing AntiFOIL, the powerful technique for factoring trinomials, we will first explain where the method comes from. We will discuss the technique itself in the next section.

B. Undoing FOIL

By FOIL, $(3x - 4)(x + 2) = 3x^2 + 6x - 4x - 8$

Without combining like terms, let's factor by grouping . . .

$$3x(x + 2) - 4(x + 2)$$

$$(x + 2)(3x - 4)$$

Thus we have $(3x - 4)(x + 2)$, which is what we started with.

Note: We've learned that we can undo FOIL (using factoring by grouping) if the middle terms are not combined.

C. Some Motivating Questions

Question: If the middle terms had been combined as $3x^2 + 2x - 8$, how could we factor it?

Partial Answer: Given $3x^2 + 2x - 8$, we rewrite it as $3x^2 + 6x - 4x - 8$ and then factor by grouping.

This answer creates the following question:

Question: How do we know that $3x^2 + 2x - 8$ should be written as $3x^2 + 6x - 4x - 8$ instead of something else?

To begin to answer this, we make the following observations:

Observation 1: $6(-4) = 3(-8)$. Will this always be true?

Observation 2: The signs of the middle terms are $+/-$.
How do we know they should be this?

D. Comments on Observations

1. Studying Observation 1

By FOIL, $(ax + b)(cx + d) = acx^2 + adx + bcx + bd$

Let $f = ac$, $o = ad$, $i = bc$, $l = bd$

Note that $fl = acbd$ and $oi = adbc$, so $fl = oi$.

We see that this formula must hold if we split the middle correctly. Thus our guess was correct. It is always true!

The formula $fl = oi$ is called the **AntiFOIL formula**

2. Regarding Observation 2: We need a rule for signs.

We define the **Trinomial Sign Pairing (TSP)** as follows:

$$\begin{array}{l|l} ax^2 + bx + c & \text{TSP} \\ ax^2 - bx + c & +, + \\ ax^2 + bx - c & -, - \\ ax^2 - bx - c & +, - \end{array}$$

In words, if the constant is positive, the signs are the same; otherwise, they alternate (for $a > 0$).

Thus, for example, the TSP for $x^2 - 5x + 6$ is $-$, $-$, and this tells us that the two middle terms should both be negative.

E. Conclusion

To factor trinomials, we put the TSP, AntiFOIL formula, factoring by grouping into a systematic strategy. This strategy is called AntiFOIL and is discussed in the next section.