5.2 Dividing Polynomials

A. Dividing Polynomials By Monomials

To divide a polynomial by a monomial, we form a fraction with the monomial in the denominator. Then divide the denominator into each term.

Example 1: Divide $6x^3 - 4x^2 + 20x$ by $2x$

Solution

$$\frac{6x^3 - 4x^2 + 20x}{2x} = \frac{6x^3}{2x} - \frac{4x^2}{2x} + \frac{20x}{2x}$$

Ans $\boxed{3x^2 - 2x + 10}$

Example 2: Divide $18a^2b - 12ab^2 + 24ab$ by $-3ab$

Solution

$$\frac{18a^2b - 12ab^2 + 24ab}{-3ab} = \frac{18a^2b}{-3ab} + \frac{-12ab^2}{-3ab} + \frac{24ab}{-3ab}$$

Ans $\boxed{-6a + 4b - 8}$

To divide polynomials by multiple-term polynomials, we need to do long division.
For comparison, we first review long division of numbers . . .

B. Review of Long Division

Consider $6951 \div 327$ and avoid decimals:

$$327 \,\overline{)\,6951} \qquad \longleftarrow \text{ To estimate 327 into 695, we guess 3 into 6, i.e. 2}$$

$$\begin{array}{r}
2 \\
327 \,\overline{)\,6951} \\
654
\end{array} \quad \longleftarrow \text{ Now multiply } 2 \cdot 327$$

$$\begin{array}{r}
2 \\
327 \,\overline{)\,6951} \\
-\underline{654} \\
41
\end{array} \quad \longleftarrow \text{ Subtract: now bring down the next term, i.e. 1}$$

$$\begin{array}{r}
2 \\
327 \,\overline{)\,6951} \\
-\underline{654} \\
411
\end{array} \quad \longleftarrow \text{ Repeat: 327 into 411, guess by taking 3 into 4, i.e. 1}$$

$$\begin{array}{r}
21 \\
327 \,\overline{)\,6951} \\
-\underline{654} \\
411 \\
327
\end{array} \quad \longleftarrow \text{ Now multiply } 1 \cdot 327$$

$$\begin{array}{r}
21 \\
327 \,\overline{)\,6951} \\
-\underline{654} \\
411 \\
-\underline{327} \\
84
\end{array} \quad \begin{array}{l} \longleftarrow \text{ Now subtract} \\ \longleftarrow \text{ Must stop, if we don't want decimals} \end{array}$$

Ans $\boxed{21\frac{84}{327}}$

C. Dividing Polynomials By Multiple Term Polynomials

We use what is called **algebraic long division**.

You do the same steps as in long division of numbers, with a few extra things to consider.

Important Extra Features:

1. Write the polynomials in descending order.

2. If any powers are missing, then **include** zero terms.
   These act as placeholders.

3. When subtracting polynomials, put parentheses around the polynomial.
   The minus sign affects everything.

4. You stop when the **degree** of the remainder is smaller than the divisor.

5. You need to separate the remainder with an extra + sign (no mixed number!)

This process is easier to do, than to describe.

Look at the following examples . . .

Example 1: Divide $3x^3 + 4x^2 + x + 7$ by $x^2 + 1$

Solution

$$x^2 + 0x + 1 \;\overline{\big)\; 3x^3 + 4x^2 + x + 7} \qquad \longleftarrow \text{ Divide } x^2 \text{ into } 3x^3\text{: } 3x$$

$$\begin{array}{rl} & 3x \\ x^2 + 0x + 1 & \overline{\big)\; 3x^3 + 4x^2 + x + 7} \\ & \;\; 3x^3 + 0x^2 + 3x \quad \longleftarrow \text{ Multiply } 3x(x^2 + 0x + 1) \\ & \qquad\qquad\qquad\qquad \text{Now subtract. Remember parentheses.} \end{array}$$

$$\begin{array}{rl} & 3x \\ x^2 + 0x + 1 & \overline{\big)\; 3x^3 + 4x^2 + x + 7} \\ & \underline{-(3x^3 + 0x^2 + 3x)} \\ & \qquad\quad 4x^2 - 2x + 7 \quad \longleftarrow \text{ Bring down 7} \\ & \qquad\qquad\qquad\qquad \text{Now divide } x^2 \text{ into } 4x^2\text{: } 4. \end{array}$$

$$\begin{array}{rl} & 3x + 4 \\ x^2 + 0x + 1 & \overline{\big)\; 3x^3 + 4x^2 + x + 7} \\ & \underline{-(3x^3 + 0x^2 + 3x)} \\ & \qquad\quad 4x^2 - 2x + 7 \\ & \qquad\quad 4x^2 + 0x + 4 \quad \longleftarrow \text{ Multiply } 4(x^2 + 0x + 1) \\ & \qquad\qquad\qquad\qquad \text{Now subtract. Remember parentheses.} \end{array}$$

$$\begin{array}{rl} & 3x + 4 \\ x^2 + 0x + 1 & \overline{\big)\; 3x^3 + 4x^2 + x + 7} \\ & \underline{-(3x^3 + 0x^2 + 3x)} \\ & \qquad\quad 4x^2 - 2x + 7 \\ & \qquad\; \underline{-(4x^2 + 0x + 4)} \\ & \qquad\qquad\quad -2x + 3 \quad \longleftarrow \text{ Stop: smaller degree than } x^2 + 1. \end{array}$$

Ans $\boxed{3x + 4 + \dfrac{-2x + 3}{x^2 + 1}}$

Example 2: Divide $8x^3 - x^2 + 4$ by $2x^2 - 3$

Solution

$$2x^2 + 0x - 3 \,\overline{\big)\, 8x^3 - x^2 + 0x + 4} \qquad \longleftarrow \text{Divide } 2x^2 \text{ into } 8x^3\text{: } 4x$$

$$\begin{array}{r} 4x \\ 2x^2 + 0x - 3 \,\overline{\big)\, 8x^3 - x^2 + 0x + 4} \\ 8x^3 + 0x^2 - 12x \end{array} \quad \longleftarrow \text{Multiply } 4x(2x^2 + 0x - 3)$$

Now subtract. Remember parentheses.

$$\begin{array}{r} 4x \\ 2x^2 + 0x - 3 \,\overline{\big)\, 8x^3 - x^2 + 0x + 4} \\ \underline{-(8x^3 + 0x^2 - 12x)} \\ -x^2 + 12x + 4 \end{array} \quad \longleftarrow \text{Bring down } 4$$

Now divide $2x^2$ into $-x^2$: $-\frac{1}{2}$.

$$\begin{array}{r} 4x - \frac{1}{2} \\ 2x^2 + 0x - 3 \,\overline{\big)\, 8x^3 - x^2 + 0x + 4} \\ \underline{-(8x^3 + 0x^2 - 12x)} \\ -x^2 + 12x + 4 \\ -x^2 + 0x + \frac{3}{2} \end{array} \quad \longleftarrow \text{Multiply } -\frac{1}{2}(2x^2 + 0x - 3)$$

Now subtract. Remember parentheses.

$$\begin{array}{r} 4x - \frac{1}{2} \\ 2x^2 + 0x - 3 \,\overline{\big)\, 8x^3 - x^2 + 0x + 4} \\ \underline{-(8x^3 + 0x^2 - 12x)} \\ -x^2 + 12x + 4 \\ \underline{-\left(-x^2 + 0x + \frac{3}{2}\right)} \\ 12x + \frac{5}{2} \end{array} \quad \longleftarrow \text{Stop: smaller degree than } 2x^2 - 3.$$

Ans $\boxed{4x - \frac{1}{2} + \frac{12x + \frac{5}{2}}{2x^2 - 3}}$