5.1B Conjugates, Square Formula, and Powers

A. Conjugates

Binomials that differ only in the sign of their second terms are called **algebraic conjugates**.

Examples of Algebraic Conjugates

$3x + 7y$ and $3x - 7y$

$4 - 3x^2$ and $4 + 3x^2$

$7m + 8n^3$ and $7m - 8n^3$

B. Multiplying Conjugates

When multiplying conjugates, the outer and inner terms will always cancel when you do FOIL.

Therefore, if you multiply conjugates you just need to do FL.

Hence, we have the **FL shortcut for conjugates**.

Examples:

1. Find $(4x + 5y)(4x - 5y)$

   By FL, we immediately get $\boxed{16x^2 - 25y^2}$

2. Find $(5 - 4x^3y)(5 + 4x^3y)$

   By FL, we immediately get $\boxed{25 - 16x^6y^2}$

C. Square Formula

1. Derivation

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

2. Square Formula

Notice that squaring gives you **three** terms.

Square formula:	1.Square the first term 2.Product times two 3.Square the last term

Squaring happens frequently. You need to know this to save time!

3. Examples

Example 1: Simplify $(4x-3y)^2$

Solution

Use the square formula!

1. Square the first term: $(4x)^2 = 16x^2$

2. Product times two: $(4x)(-3y)\cdot 2 = -24xy$

3. Square the last term: $(-3y)^2 = 9y^2$

Ans $\boxed{16x^2 - 24xy + 9y^2}$

Example 2: Simplify $(3x^2 + 5)^2$

Solution

Use the square formula!

1. Square the first term: $(3x^2)^2 = 9x^4$
2. Product times two: $(3x^2)(5) \cdot 2 = 30x^2$
3. Square the last term: $(5)^2 = 25$

Ans $\boxed{9x^4 + 30x^2 + 25}$

Example 3: Simplify $(7m^2n - 4mn^3)^2$

Solution

Use the square formula!

1. Square the first term: $(7m^2n)^2 = 49m^4n^2$
2. Product times two: $(7m^2n)(-4mn^3) \cdot 2 = -56m^3n^4$
3. Square the last term: $(-4mn^3)^2 = 16m^2n^6$

Ans $\boxed{49m^4n^2 - 56m^3n^4 + 16m^2n^6}$

Severe Warning: DON'T forget the middle term in the square formula!

D. Powers

Fundamental Rule:

You **can't** apply powers across $+$ or $-$ signs directly. The square formula applies!

General Strategy:

Break power up into products of squares. Use the square formula, followed by the factor table.

Examples

Example 1: Simplify $(2x - 3y)^3$

Solution

Write $(2x - 3y)^3$ as $(2x - 3y)^2(2x - 3y)$

Use the square formula!

1. Square the first term: $(2x)^2 = 4x^2$
2. Product times two: $(2x)(-3y) \cdot 2 = -12xy$
3. Square the last term: $(-3y)^2 = 9y^2$

Thus we have $(4x^2 - 12xy + 9y^2)(2x - 3y)$

Now use the factor table.

	$4x^2$	$-12xy$	$+9y^2$
$2x$	$8x^3$	$-24x^2y$	$18xy^2$
$-3y$	$-12x^2y$	$36xy^2$	$-27y^3$

Ans $\boxed{8x^3 - 36x^2y + 54xy^2 - 27y^3}$

Example 2: Simplify $(5a^2b + 4ab^2)^3$

Solution

Write $(5a^2b + 4ab^2)^3$ as $(5a^2b + 4ab^2)^2(5a^2b + 4ab^2)$

Use the square formula!

Thus we have $(25a^4b^2 + 40a^3b^3 + 16a^2b^4)(5a^2b + 4ab^2)$

Now use the factor table.

	$25a^4b^2$	$+40a^3b^3$	$+16a^2b^4$
$5a^2b$	$125a^6b^3$	$200a^5b^4$	$90a^4b^5$
$+4ab^2$	$100a^5b^4$	$160a^4b^5$	$64a^3b^6$

Ans $\boxed{125a^6b^3 + 300a^5b^4 + 250a^4b^5 + 64a^3b^6}$

Example 3: Simplify $(x-2y)^4$

Solution

Write $(x-2y)^4$ as $(x-2y)^2(x-2y)^2$

Use the square formula!

Thus we have $(x^2-4xy+4y^2)(x^2-4xy+4y^2)$

Now use the factor table.

	x^2	$-4xy$	$+4y^2$
x^2	x^4	$-4x^3y$	$4x^2y^2$
$-4xy$	$-4x^3y$	$16x^2y^2$	$-16xy^3$
$+4y^2$	$4x^2y^2$	$-16xy^3$	$16y^4$

Ans $\boxed{x^4-8x^3y+24x^2y^2-32xy^3+16y^4}$