5.1A Polynomials: Basics

A. Definition of a Polynomial

A **polynomial** is a combination of terms containing numbers and variables raised to positive (or zero) whole number powers.

Examples of Polynomials

$3x^5y^3 - 4xy^4 + 5x^2y - 6$

$4x^2 + 2x^5 - 7$

NOT polynomials

$2x^{\frac{1}{2}} + 4$ (power is a fraction)

$4x^{-1} - 2x^3$ (power is negative)

B. Terminology

1. **Degree**

a. Term Degree: sum of powers in a term

x^5y^3	the degree is 8
xy^2z	the degree is 4
4	the degree is 0

b. Polynomial Degree: maximum (not total) term degree

$3x^5y^3 - 4xy^4 + 5x^2y - 6$ the degree is 8

$4x^2 + 2x^5 - 7$ the degree is 5

2. **Descending Order**

We often write polynomials in order from the highest term degree to the the lowest.

For instance, we rewrite $4x^2 + 2x^5 - 7$ as $2x^5 + 4x^2 - 7$

C. Adding/Subtracting Polynomials

We combine like terms as before.

Beware: minus signs and parentheses

1. Find $(6x^2 - 4x - 3) + (3x^2 + 4)$

$\boxed{9x^2 - 4x + 1}$

2. Find $(5x^3 + 3x - 2) - (6x^2 - 4x + 2)$

$5x^3 + 3x - 2 - 6x^2 + 4x - 2 = \boxed{5x^3 - 6x^2 + 7x - 4}$

D. Multiplying Polynomials By Monomials

A **monomial** is a one-term polynomial. Use the distributive property.

Find $6x^3y(2xy - 7x + 8y^2)$

$\boxed{12x^4y^2 - 42x^4y + 48x^3y^3}$

E. Multiplying Binomials

A **binomial** is a two-term polynomial.

Method 1: Distributive Property

If the problem is to expand $(6x - 4y)(x^2 + 3y)$, we distribute the $(6x - 4y)$ to the two terms of the second binomial:

$$(6x - 4y)(x^2 + 3y) = (6x - 4y)x^2 + (6x - 4y)3y$$

Now use the distributive property again to get $\boxed{6x^3 - 4x^2y + 18xy - 12y^2}$

A shortcut to the above method is called **FOIL**

Method 2: FOIL

FOIL is an acronym for “First-Outer-Inner-Last”

Consider the following example:

Find $(5x + 2y)(6x - y)$ using FOIL

First: $(5x)(6x) = 30x^2$

Outer: $(5x)(-y) = -5xy$

Inner: $(2y)(6x) = 12xy$

Last: $(2y)(-y) = -2y^2$

Thus we get $30x^2 - 5xy + 12xy - 2y^2 = \boxed{30x^2 + 7xy - 2y^2}$

F. Multiplying Polynomials of Any Size

Method 1: Distributive Property

If the problem is to expand $(3x^2 - 4x + 4)(x^2 + 2x - 3)$, we distribute the $(3x^2 - 4x + 4)$ to the terms of the second polynomial:

$$(3x^2 - 4x + 4)(x^2 + 2x - 3)$$

$$= (3x^2 - 4x + 4)x^2 + (3x^2 - 4x + 4)2x + (3x^2 - 4x + 4)(-3)$$

Now use the distributive property again

$$3x^4 - 4x^3 + 4x^2 + 6x^3 - 8x^2 + 8x - 9x^2 + 12x - 12$$

Thus, after combining like terms, we get $\boxed{3x^4 + 2x^3 - 13x^2 + 20x - 12}$

A shortcut to the above method is called the **factor table**

Method 2: Factor Table

You make a “tic-tac-toe” grid, and fill in the boxes with the products.

Consider $(2x^2y - 4y^2 + x^2)(3xy + xy^2 - 4)$

Make factor table:

	$2x^2y$	$-4y^2$	x^2
$3xy$			
$+xy^2$			
-4			

Then fill in the table with the products:

	$2x^2y$	$-4y^2$	$+x^2$
$3xy$	$+6x^3y^2$	$-12xy^3$	$+3x^3y$
$+xy^2$	$+2x^3y^3$	$-4xy^4$	$+x^3y^2$
-4	$-8x^2y$	$+16y^2$	$-4x^2$

Collecting like terms:

$$\boxed{7x^3y^2 - 12xy^3 + 3x^3y + 2x^3y^3 - 4xy^4 - 8x^2y + 16y^2 - 4x^2}$$