4.3 Applications

A. Comments

Sometimes when we solve word problems,
we end up with a system of linear equations to solve.

B. Examples

Example 1: 372 people attended a concert. Floor seats cost \$20/each. Balcony seats cost \$12 each. If the ticket sales receipts totaled \$6144, how many tickets of each type were sold?

Solution

1. 372 people attend
   Floor seats \$20/each
   Balcony seats \$12/each
   Total sales $= \$6144$
   How many tickets sold (for each type)?

2. Let x =number of floor seats sold
   Let y =number of balcony seats sold.
   Then $x + y = 372$. Now need another equation!
   $20x$ =money from floor seats
   $12y$ =money from balcony seats
   Total money earned= $20x + 12y = 6144$

3. Thus we have the system $\begin{cases} x + y = 372 & (1) \\ 20x + 12y = 6144 & (2) \end{cases}$

4. Solve the system by substitution:

Solve (1) for y: $\quad y = 372 - x$

Substitute into (2):

$$20x + 12(372 - x) = 6144$$

$$20x + 4464 - 12x = 6144$$

$$8x + 4464 = 6144$$

$$8x = 1680$$

$$x = 210$$

so $\quad y = 372 - 210 = 162.$

Hence we have that $(x, y) = (210, 162)$.

Ans

210 floor seats 162 balcony seats

Example 2: An airplane travels between two cities that are 1200 miles apart.
The trip against the wind takes 2 hours and 40 minutes.
The return trip (with the wind) takes $2\frac{1}{2}$ hours.
What is the speed of the plane in still air?
What is the speed of the wind?

Solution

1. Airplane trip 1200 miles

   Against wind; trip $2\frac{40}{60} = 2\frac{2}{3} = \frac{8}{3}$ hours

   With wind; trip $2\frac{1}{2} = \frac{5}{2}$ hours

   Find airplane speed and wind speed.

2. Let x =airplane speed (in still air)
   Let y =wind speed
   Then $x + y \implies$ traveling speed with wind
   Also $x - y \implies$ traveling speed against wind
   "Distance, Rate, Time Problem" $d = rt$

3. Thus we have the system $\begin{cases} 1200 = (x+y)\left(\frac{5}{2}\right) \\ 1200 = (x-y)\left(\frac{8}{3}\right) \end{cases}$

$$\begin{cases} \frac{5}{2}(x+y) = 1200 \\ \frac{8}{3}(x-y) = 1200 \end{cases}$$

Clearing parentheses:

$$\begin{cases} \frac{5}{2}x + \frac{5}{2}y = 1200 \\ \frac{8}{3}x + \frac{8}{3}y = 1200 \end{cases}$$

Clearing fractions:

$$\begin{cases} 5x + 5y = 2400 & (1) \\ 8x - 8y = 3600 & (2) \end{cases}$$

Multiplying eqn (1) by 8 and eqn (2) by 5:

$$\begin{cases} 40x + 40y = 19200 \\ 40x - 40y = 18000 \end{cases}$$

Adding, we get $80x = 37200 \implies x = \dfrac{37200}{80} = 465$

Substituting $x = 465$ into eqn (1), we get:

$$5(465) + 5y = 2400$$

$$2325 + 5y = 2400 \implies 5y = 75 \implies y = 15$$

Hence, $(x, y) = (465, 15)$.

Ans

speed of plane in still air: 465mph speed of wind: 15mph