3.6 Graphing Functions

A. Graphing Functions

To graph a function, we make a table of (x, y) pairs.
Then plot points and connect.

Note: Only graphs of first powers are "straight".

B. Examples

Example 1: Graph f, where $f(x) = x^2 + 1$

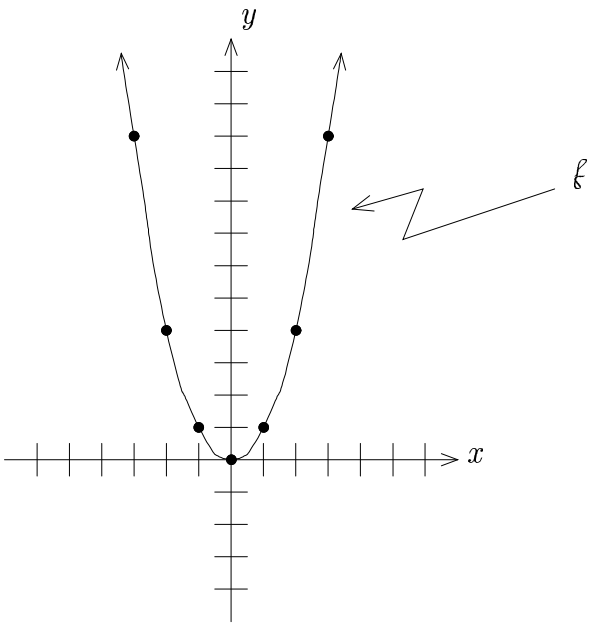
Solution

x	$y = f(x)$
-3	$(-3)^2 + 1 = 9 + 1 = 10$
-2	$(-2)^2 + 1 = 4 + 1 = 5$
-1	$(-1)^2 + 1 = 1 + 1 = 2$
0	$0^2 + 1 = 0 + 1 = 1$
1	$1^2 + 1 = 1 + 1 = 2$
2	$2^2 + 1 = 4 + 1 = 5$
3	$3^2 + 1 = 9 + 1 = 10$

Plot the points:

$$(-3, 10),\ (-2, 5),\ (-1, 2),\ (0, 1),\ (1, 2),\ (2, 5),\ (3, 10)$$

Then connect the dots in a smooth curve:



Example 2: Graph g, where $g(x) = -|x+2|$

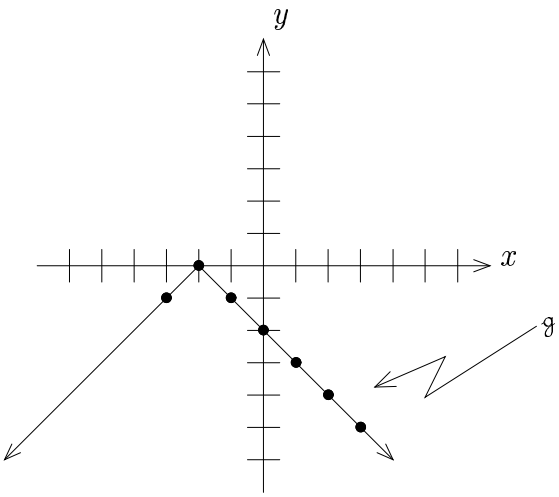
Solution

x	$y = f(x)$
-3	$-\|(-3)+2\| = -\|-1\| = -(1) = -1$
-2	$-\|(-2)+2\| = -\|0\| = -(0) = 0$
-1	$-\|(-1)+2\| = -\|1\| = -1$
0	$-\|0+2\| = -\|2\| = -2$
1	$-\|1+2\| = -\|3\| = -3$
2	$-\|2+2\| = -\|4\| = -4$
3	$-\|3+2\| = -\|5\| = -5$

Plot the points:

$$(-3,-1),\ (-2,0),\ (-1,-1),\ (0,-2),\ (1,-3),\ (2,-4),\ (3,-5)$$

Then connect the dots:



C. Reciprocal Function

A special function that takes some care to graph is
the function f, given by $f(x) = \dfrac{c}{x}$.

i.e. $\quad f(x) = \dfrac{3}{x}, \quad f(x) = -\dfrac{2}{x}, \quad f(x) = \dfrac{7}{x}, \quad$ etc.

This is called the **reciprocal function**.

Features:

1. The graph comes in "two pieces".

2. The graph does not cross the x axis or y axis anywhere.

3. The function is not defined for $x = 0$.

4. The curve "approaches" but does not touch either axis.

Example: Graph f, where $f(x) = \dfrac{3}{x}$

Solution

x	$y = f(x)$
-3	$\dfrac{3}{-3} = -1$
-2	$\dfrac{3}{-2} = -\dfrac{3}{2}$
-1	$\dfrac{3}{-1} = -3$
$-\dfrac{1}{2}$	$\dfrac{3}{-\frac{1}{2}} = -6$
0	undefined
$\dfrac{1}{2}$	$\dfrac{3}{\frac{1}{2}} = 6$
1	$\dfrac{3}{1} = 3$
2	$\dfrac{3}{2} = \dfrac{3}{2}$
3	$\dfrac{3}{3} = 1$

Plot the points:

$$(-3,-1),\ \left(-2,-\frac{3}{2}\right),\ (-1,-3),\ \left(-\frac{1}{2},6\right),\ \left(\frac{1}{2},6\right),\ (1,3),\ \left(2,\frac{3}{2}\right),\ (3,1)$$

Then connect the dots in smooth curve pieces: