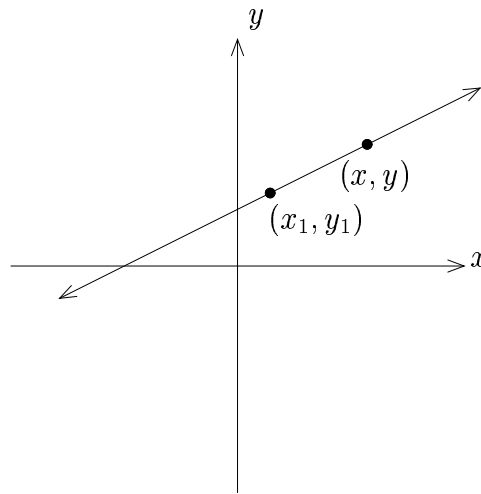


3.3 Equations of Lines

A. Point-Slope Formula

1. **Formula:** $y - y_1 = m(x - x_1)$

2. **Justification:**



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

Multiplying by $x - x_1$ yields the equation.

B. Slope-Intercept Formula

1. **Formula:** $y = mx + b$

Here “ b ” is the y -intercept.

2. **Justification:**

$(0, b)$ lies on the line, so plug $(0, b)$ in for (x_1, y_1) in the point-slope formula:

$$y - b = m(x - 0) \implies y = mx + b$$

C. Standard Form

1. **Formula:** $Ax + By = C$

2. The formula includes horizontal and vertical lines.

3. **Solving the formula for y :**

$$By = -Ax + C$$

$$y = \frac{-Ax + C}{B} \quad \text{if } B \neq 0 \quad [B = 0 \text{ vertical line}]$$

$$y = \left(-\frac{A}{B}\right)x + \left(\frac{C}{B}\right)$$

4. **Slope:** $m = -\frac{A}{B}$

D. General Strategy for Finding the Equation of a Line

1. Find the slope of the given line.
 - a. If the line is horizontal, $m = 0$
 - b. If the line is vertical, m is undefined.
 - c. Given **two points**, use $m = \frac{y_2 - y_1}{x_2 - x_1}$
 - d. Given a **line in slope-intercept form**, i.e. $y = mx + b$: read off m
 - e. Given a **line in standard form**, i.e. $Ax + By = C$: use $m = -\frac{A}{B}$

2. Find the slope of the line you want.
 - a. If you want a parallel line, then
 - I. parallel to a horizontal line is a horizontal line!
Answer to problem is $y = b$. YOU'RE DONE!
 - II. parallel to a vertical line is a vertical line!
Answer to problem is $x = a$. YOU'RE DONE!
 - III. otherwise take the same slope from step 1
 - b. If you want a perpendicular line, then
 - I. perpendicular to a horizontal line is a vertical line!
Answer to problem is $x = a$. YOU'RE DONE!
 - II. perpendicular to a vertical line is a horizontal line!
Answer to problem is $y = b$. YOU'RE DONE!
 - III. otherwise take the negative reciprocal of the slope from step 1

3. Use the new slope and the chosen point in the point-slope formula.
4. To put the answer in:
 - a. Point-slope: YOU'RE DONE!
 - b. Slope-Intercept: Solve the equation for y and simplify.
 - c. Standard Form:
 - I. First get in slope-intercept form
 - II. Get all the x 's and y 's on one side and simplify.
 - III. (OPTIONAL) Clear fractions, making x coefficient positive.

E. Examples

Example 1: Find the equation of the line with slope -2 passing through $(-3, 4)$. Write the equation in standard form.

Solution

Point-Slope:

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - (-3))$$

$$y - 4 = -2(x + 3)$$

Now put in standard form:

$$y - 4 = -2x - 6$$

$$2x + y - 4 = -6$$

Ans $\boxed{2x + y = -2}$

Example 2: Find the equation of the line passing through $(-3, 2)$ and $(1, 4)$. Write the equation in slope-intercept form.

Solution

Find slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{1 - (-3)} = \frac{2}{4} = \frac{1}{2}$$

Point-slope:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{1}{2}(x - (-3)) \quad (\text{also works if you use } (1, 4))$$

$$y - 2 = \frac{1}{2}(x + 3)$$

Now put in slope-intercept form: solve for y

$$y - 2 = \frac{1}{2}x + \frac{3}{2}$$

$$y = \frac{1}{2}x + \frac{3}{2} + 2$$

Ans $\boxed{y = \frac{1}{2}x + \frac{7}{2}}$

Example 3: Find the equation of the line passing through $(-1, 4)$ and $(-1, 7)$

Solution

Find slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{-1 - (-1)} = \frac{3}{-1 + 1} = \frac{3}{0} \quad \text{undefined}$$

vertical line! $x = a!$

Ans $\boxed{x = -1}$

Example 4: Find the equation of the line passing through $(1, -3)$ and perpendicular to $3x - 2y = 5$. Write your answer in point-slope form.

Solution

Find slope:

$$m = -\frac{A}{B} = -\frac{3}{-2} = \frac{3}{2}$$

Perpendicular slope: $m_{\perp} = -\frac{2}{3}$

Point-Slope:

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = -\frac{2}{3}(x - 1)$$

Ans $\boxed{y + 3 = -\frac{2}{3}(x - 1)}$

Example 5: Find the equation of the line passing through $(-2, 4)$ that is perpendicular to the line passing through $(3, -4)$ and $(3, 7)$

Solution

Find slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-4)}{3 - 3} = \frac{11}{0} \text{ undefined}$$

vertical line! perpendicular to a vertical line is a horizontal line
A horizontal line has equation $y = b$

Ans $\boxed{y = 4}$