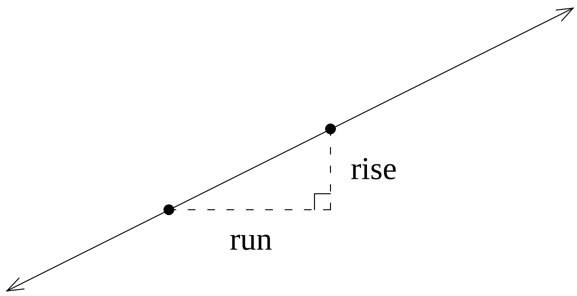
3.2 Slope

A. Slope

The slope of a line measures "how much it is tilting".



By definition, $\boxed{\text{slope} = \dfrac{\text{rise}}{\text{run}}}$.

B. Finding Slope

1. Pick two points on a line.
   Pick one as the first point and the other as the second point.

2. Going from the first point to the second, record the vertical change–the **rise**, and record the horizontal change–the **run**.

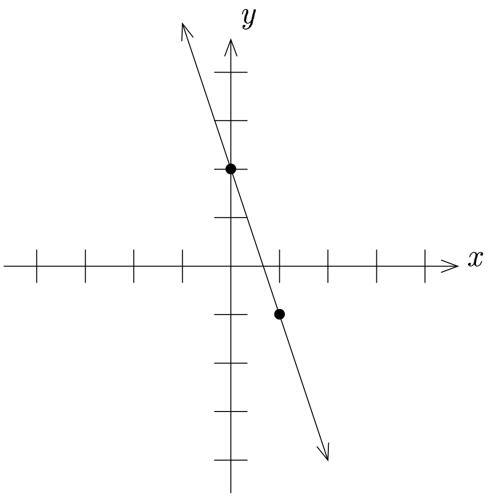
Note: These changes can be negative.

3. Use $\text{slope} = \dfrac{\text{rise}}{\text{run}}$.

Note: It doesn't matter which point you use for the first point.

C. Example

Find the slope of the given line:



Solution

Picking the top point as the first point:

Going to the bottom point:

rise$= -3$

run$= 1$

slope$= \frac{\text{rise}}{\text{run}} = \frac{-3}{1} = -3$

Note that you would get the same answer
if you chose the bottom point as the first point:

Going to the top point:

rise$= 3$

run$= -1$

slope$= \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{-1} = -3$

D. Comments on Slope

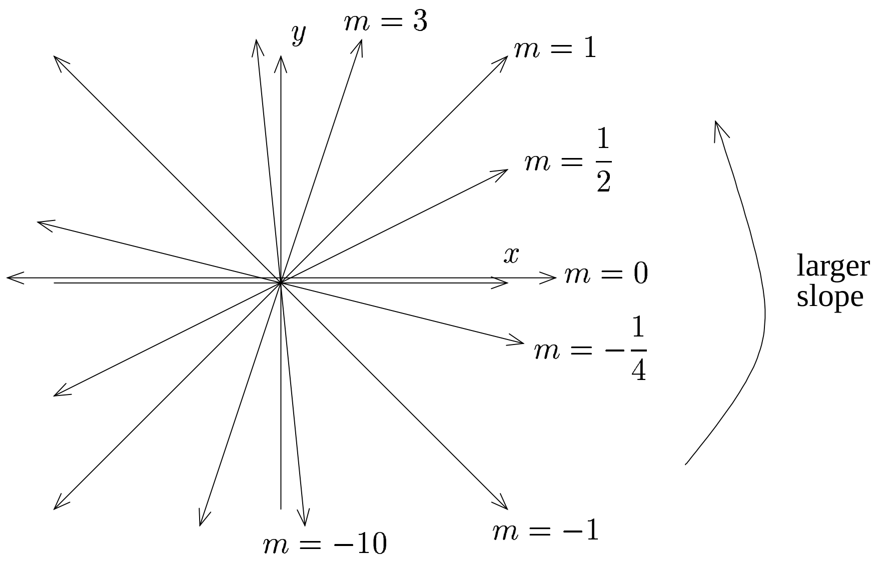
1. As in the previous example, slope can be negative!

2. Slope of a horizontal line $= 0$ $\left[\dfrac{\text{rise}}{\text{run}} = \dfrac{0}{\text{run}} = 0\right]$

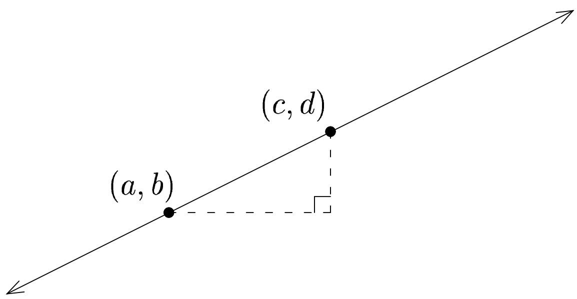
   Slope of a vertical line is **undefined** $\left[\dfrac{\text{rise}}{\text{run}} = \dfrac{\text{rise}}{0} : \text{ undefined}\right]$

3. The slope of a line is given the symbol m.

4. General idea of the slope of a line:



E. Slope Formula–Intro



rise= $d - b$

run= $c - a$

Thus, $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{d - b}{c - a}$

F. Subscripts

Four letters, as used in the previous formula, is confusing!

To help memory, we call the x and y coordinates of a point as "x" and "y", but we need to distinguish which point we mean.

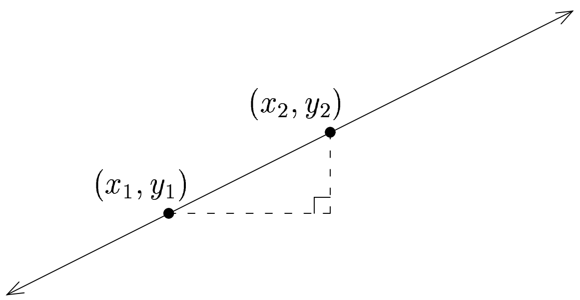
Introduce **subscripts** (small numbers below):

(x_1, y_1) first point

(x_2, y_2) second point

The subscripts are only **labels** and don't have any "value".

G. Slope Formula



rise= $y_2 - y_1$
run= $x_2 - x_1$

Thus, $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. This is our "two-point formula" for slope:

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

H. Example

Find the slope of the line passing through $(-1, 3)$ and $(4, -2)$

Solution

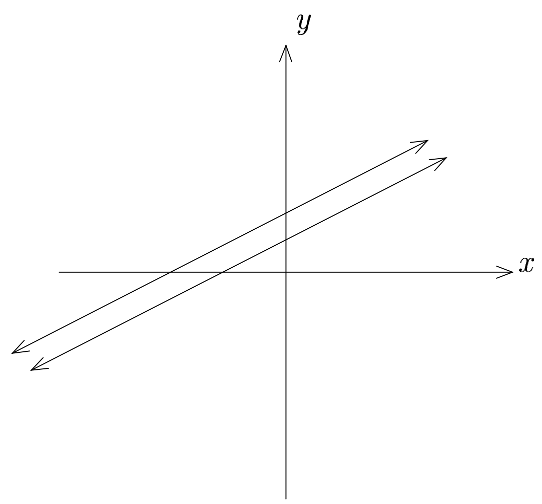
It doesn't matter which point we pick for the first point.

Call $(x_1, y_1) = (-1, 3)$ and call $(x_2, y_2) = (4, -2)$.

Now use the "two-point formula":

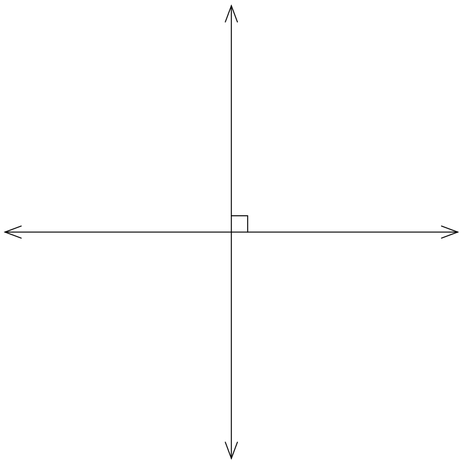
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-2) - (3)}{(4) - (-1)} = \frac{-5}{5} = \boxed{-1}$$

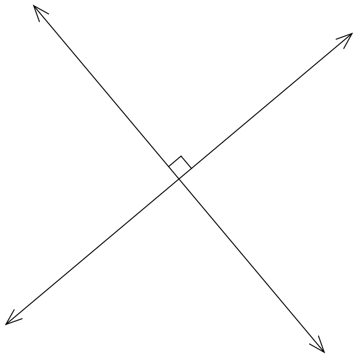
I. Parallel Lines



1. Horizontal lines are parallel to each other.
2. Vertical lines are parallel to each other.
3. All other angled lines have the **same slope**: $m_1 = m_2$
4. Given a line with slope m, we call the parallel slope: $m_{||}$ (same number!)

J. Perpendicular Lines





1. Horizontal and vertical lines are **perpendicular** to each other.

2. All other angled lines are perpendicular if and only if their slopes are **negative (or opposite) reciprocals**.

$$m_2 = -\frac{1}{m_1} \quad \text{and} \quad m_1 = -\frac{1}{m_2}$$

3. Given a line with slope m, we call the perpendicular slope: $m_\perp$.

4. **Examples:** What is the slope of the line perpendicular to a line with slope

a. 4 ?

b. $-\frac{2}{3}$?

c. undefined?

Solution

a. $\boxed{m_{\perp} = -\frac{1}{4}}$

b. $-\frac{1}{-\frac{2}{3}} \implies \boxed{m_{\perp} = \frac{3}{2}}$

c. perpendicular to a vertical line is a horizontal line $\implies \boxed{m_{\perp} = 0}$