2.8 Absolute Value Inequalities

A. Strategy

Recall: Absolute Value means **distance from the origin**

1. Draw a number line and mark the locations that have the required distance. Remember distance is always a “positive” idea.

2. Use the shaded number line you just drew to rewrite the original problem without absolute value signs. You will typically get one inequality for each endpoint.

3. Solve the resulting problem, usually a compound inequality.

Note: Most of the time, the problem breaks into two inequalities joined by AND/OR

Guidelines:

1. AND/OR choice:

   a. $<, \leq$: AND

   b. $>, \geq$: OR

2. Distance picture

   a. $<, \leq$: “sandwich” **IF number on opposite side is positive!!**

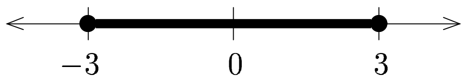
   b. $>, \geq$: “two pieces going out” **IF number on opp. side is positive!!**

B. Examples

Example 1: Solve $|2x-1| \leq 3$ for x

Solution

1. Draw the distance picture on the number line (want distance ≤ 3)



Note: When drawing the distance picture, distance doesn't "notice" the negative signs.

2. From the picture, we see that we have:

Left endpoint: $2x-1 \geq -3$

Right endpoint: $2x-1 \leq 3$

Both must be true to be inside the region: need **AND**

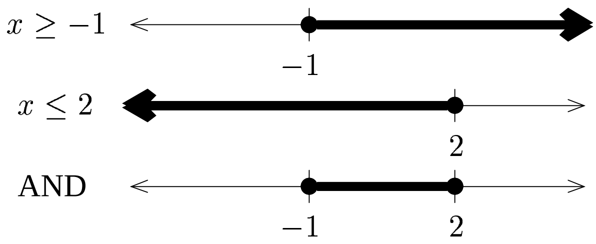
3. Now solve $2x-1 \geq -3$ AND $2x-1 \leq 3$

a. $2x \geq -2$ AND $2x \leq 4$

$\frac{2x}{2} < \frac{-2}{2}$ AND $\frac{2x}{2} \leq \frac{4}{2}$

$x \geq -1$ AND $x \leq 2$

b. Graph:



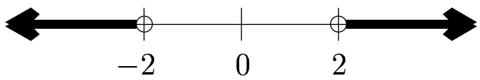
c. $-1 \leq x \leq 2$

Ans $\boxed{-1 \leq x \leq 2}$

Example 2: Solve $|4 - x| > 2$ for x

Solution

1. Draw the distance picture on the number line (want distance > 2)



Note: When drawing the distance picture, distance doesn't "notice" the negative signs.

2. From the picture, we see that we have:

Left endpoint: $4 - x < -2$

Right endpoint: $4 - x > 2$

To be in the region, only one must be true: need **OR**

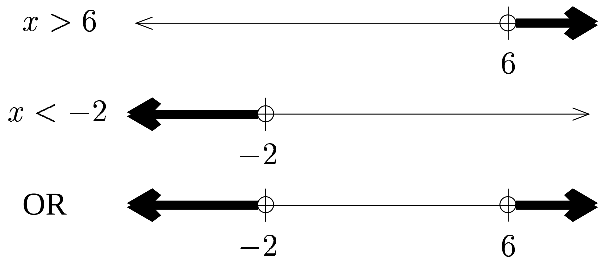
3. Now solve $4 - x < -2$ OR $4 - x > 2$

a. $-x < -6$ OR $-x > -2$

$$\frac{-x}{-1} > \frac{-6}{-1} \qquad \text{OR} \qquad \frac{-x}{-1} < \frac{-2}{2}$$

$$x > 6 \qquad \text{OR} \qquad x < -2$$

b. Graph:



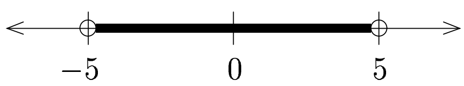
c. can not be simplified!

Ans $\boxed{x > 6 \text{ OR } x < -2}$

Example 3: Solve $|3 - 2x| < 5$ for x

Solution

1. Draw the distance picture on the number line (want distance < 5)



2. From the picture, we see that we have:

Left endpoint: $3 - 2x > -5$

Right endpoint: $3 - 2x < 5$

Both must be true to be inside the region: need **AND**

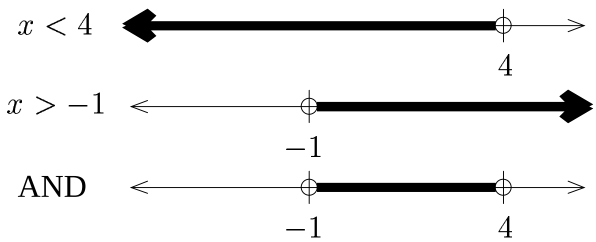
3. Now solve $3 - 2x > -5$ AND $3 - 2x < 5$

a. $-2x > -8$ AND $-2x < 2$

$$\frac{-2x}{-2} < \frac{-8}{-2} \quad \text{AND} \quad \frac{-2x}{-2} > \frac{2}{-2}$$

$$x < 4 \quad \text{AND} \quad x > -1$$

b. Graph:



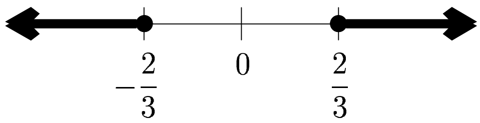
c. $-1 < x < 4$

Ans $\boxed{-1 < x < 4}$

Example 4: Solve $\left|\dfrac{5x}{6}-\dfrac{1}{4}\right| \geq \dfrac{2}{3}$ for x

Solution

1. Draw the distance picture on the number line (want distance $\geq \dfrac{2}{3}$)



2. From the picture, we see that we have:

   Left endpoint: $\dfrac{5x}{6}-\dfrac{1}{4} \leq -\dfrac{2}{3}$

   Right endpoint: $\dfrac{5x}{6}-\dfrac{1}{4} \geq \dfrac{2}{3}$

   To be in the region, only one must be true: need **OR**

3. Now solve $\dfrac{5x}{6}-\dfrac{1}{4} \leq -\dfrac{2}{3}$ OR $\dfrac{5x}{6}-\dfrac{1}{4} \geq \dfrac{2}{3}$

   a. $12\left(\dfrac{5x}{6}-\dfrac{1}{4}\right) \leq 12\left(-\dfrac{2}{3}\right)$ OR $12\left(\dfrac{5x}{6}-\dfrac{1}{4}\right) \geq 12\left(\dfrac{2}{3}\right)$

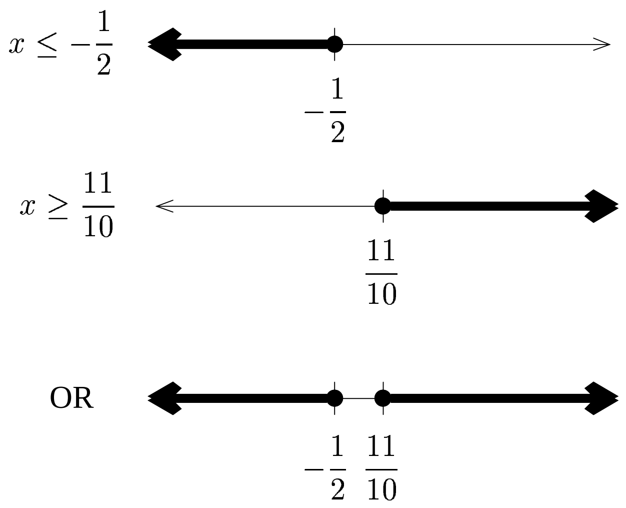
   $10x-3 \leq -8$ OR $10x-3 \geq 8$

   $10x \leq -5$ OR $10x \geq 11$

   $\dfrac{10x}{10} \leq \dfrac{-5}{10}$ OR $\dfrac{10x}{10} \geq \dfrac{11}{10}$

   $x \leq -\dfrac{1}{2}$ OR $x \geq \dfrac{11}{10}$

b. Graph:



c. can not be simplified!

Ans $\boxed{x \leq -\frac{1}{2} \text{ OR } x \geq \frac{11}{10}}$

C. Comments

1. Always draw the number line and make the "distance" picture.
   Don't just use some "rule".
   In all of the above problems, the right hand side was **positive**.
   If the right hand side is zero or negative, the solution strategy will be different **but still based on the distance idea**.

2. As per comment #1 above, think about how to solve:

   a. $|6x - 1| > -3$
   b. $|2 - 5x| \leq 0$
   c. $|7 - x| \leq -5$
   d. $|2x + 3| > 0$
   etc.