2.7B   Triple Inequalities; More AND/OR

A.   Triple Inequalities

A triple inequality is an "object" like $-3 \le x \le 2$

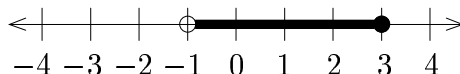**Meaning:**   We want all numbers **between** -3 and 2

Graph: "Sandwich"

$$-3 \qquad 2$$

Example: Graph $-1 < x \le 3$

Solution

Numbers between -1 and 3:   open circle at -1!

$$-4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4$$

B.   Comments on Triple Inequalities

1.   The "jaws" of the inequalities in a triple inequality all face to the right.

2.   The smaller number in a triple inequality must always be on the left.

3.   Thus the following are "illegal":

    a. $-3 \le x > 2$ (violates rule 1:   jaws must face right)

    b. $8 < x \le 1$ (violates rule 2:   smaller number on the left)

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C. Solving Inequalities with AND or OR

Given two inequalities joined by AND or OR, we

1. Graph each on a number line, obtaining two shaded number lines.

2. Find the corresponding AND or OR graph (a third line).

3. Convert back: Write the answer as an inequality or triple inequality.

Note: One type of problem yielding an answer that looks like
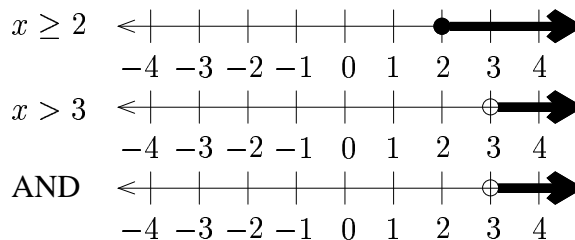
 (two pieces going out)
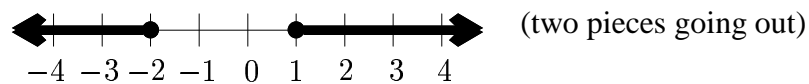
can not be simplified.

In this case, the "answer" is just the statement again of the problem.

D. Examples
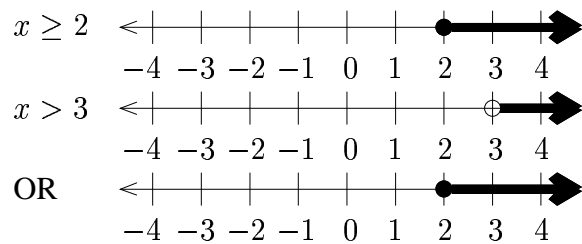
Example 1:   Find $x \geq 2$ AND $x > 3$

Solution

Ans $\boxed{x > 4}$

Example 2: Find $x \geq 2$ OR $x > 3$

Solution

$x \geq 2$
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$x > 3$
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

OR
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

Ans $\boxed{x \geq 2}$

Example 3: Find $x > -2$ AND $x \leq 3$

Solution

$x > -2$
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

$x \leq 3$
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

AND
$$-4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$
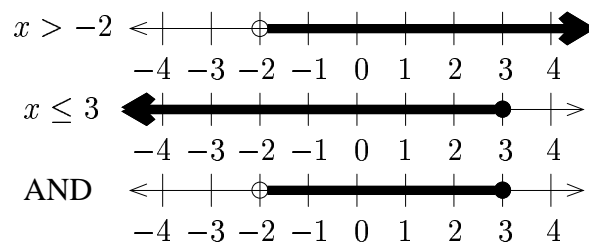
"sandwich" $\implies$ triple inequality

Ans $\boxed{-2 < x \leq 3}$

Example 4: Find $x > -2$ OR $x \leq 3$

Solution

$x > -2$

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

$x \leq 3$

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

OR

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$
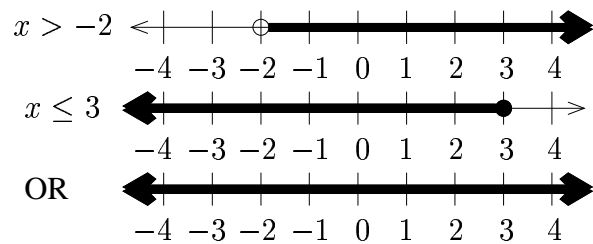
all real numbers!

Ans $\boxed{(-\infty, \infty)}$

Example 5: Find $x \leq -1$ AND $x \geq 0$

Solution

$x \leq -1$

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

$x \geq 0$

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

AND

$-4 \; -3 \; -2 \; -1 \;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$
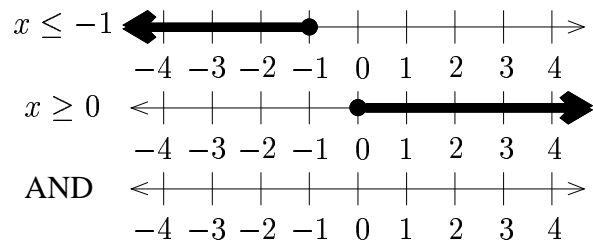
nothing!

Ans $\boxed{\emptyset}$

Example 6:   Find $x \leq -1$ OR $x \geq 0$

Solution

$x \leq -1$

$$\longleftarrow\!\!\!\!\!\!\bullet\!\!\!\!\!\!\!\text{———}\!\!\!\!\!\!\!\bullet\quad\quad\quad\longrightarrow$$

$$-4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4$$

$x \geq 0$

$$\longleftarrow\quad\quad\quad\quad\bullet\!\!\!\text{———}\!\!\!\longrightarrow$$

$$-4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4$$

OR

$$\longleftarrow\!\!\!\bullet\quad\quad\bullet\!\!\!\text{———}\!\!\!\longrightarrow$$

$$-4 \ -3 \ -2 \ -1 \ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4$$
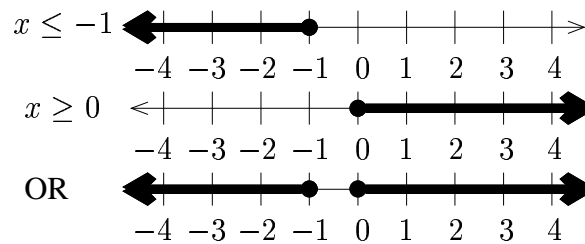
two pieces going out $\Longrightarrow$ can't simplify!

Ans $\boxed{x \leq -1 \ \text{ OR } \ x \geq 0}$

E.   Comments

1.   The six examples above demonstrate "all" of the possible combinations. Only one of the six types can't be simplified (in this class).

2.   Inequalities joined in AND or OR are called **compound inequalities**.