2.6 Linear Inequalities

A. Review of Number Inequalities

$<, >$ less than, greater than

$\leq, \geq$ less than **or** equal to, greater than **or** equal to

Examples:

$6 > 3$ [6 is bigger than 3]

$-2 \leq 4$ [-2 is less than 4 **or** equal to 4: it's less than!]

$-5 \geq -5$ [-5 is greater than -5 **or** equal to -5: it's equal to!]

To compare fractions: Find a common denominator

Example: Is $\frac{3}{4} < \frac{5}{6}$ or $\frac{3}{4} > \frac{5}{6}$?

Solution

$\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$, so since $\frac{9}{12} < \frac{10}{12}$, we have

Ans $\boxed{\frac{3}{4} < \frac{5}{6}}$

B. Graphing Inequalities on a Number Line

$>$: **open** circle, fill in to the **right**

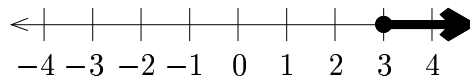$\geq$: **closed** circle, fill in to the **right**

$<$: **open** circle, fill in to the **left**

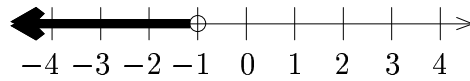$\leq$: **closed** circle, fill in to the **left**

Note: These are the rules when the variable is on the **left**.

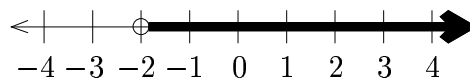**Examples**

1. $x \geq 3$

2. $x < 1$

3. $x > -2$

C. Reflections

If the variable is on the right in an inequality, the graphing rules don't work so we must first "flip over" the inequality **before** graphing:
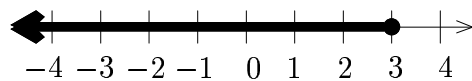
$$6 \leq x \implies x \geq 6$$

D. Additional Examples

Example 1: Graph $3 \geq x$

Solution

First flip over: $x \leq 3$

Then use rules for graphing:
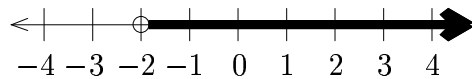

Example 2: Graph $-2 < x$

Solution

First flip over: $x > -2$

Then use rules for graphing:

E. Solving Linear Inequalities

We solve these using the same techniques as before, **except** the inequality sign switches direction if we **multiply or divide** by a **negative** number.

Example: Solve $3(5 - x) < 2x + 4$ for x **and** graph the solution

Solution

Simplify:

$$3(5 - x) < 2x + 4$$

$$15 - 3x < 2x + 4 \qquad \text{[distribute]}$$

Now isolate:

$$15 - 3x \underline{-2x} < 2x + 4 \underline{-2x}$$

$$15 - 5x < 4$$
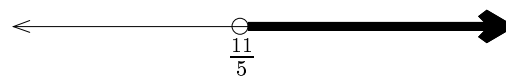
$$15 - 5x \underline{-15} < 4 \underline{-15}$$

$$-5x < -11$$

Dividing by -5 switches the inequality:

$$\frac{-5x}{-5} > \frac{-11}{-5}$$

$$\boxed{x > \frac{11}{5}}$$

Graph:

F. Comments

1. Unlike when solving equations, the answer for an inequality must have the variable on the left, such as $x < 3$.

2. If when solving an inequality, you reach an answer having the variable on the right, you must "flip over" the inequality, as discussed previously.

3. As before, only graph an inequality after you've converted it to have the variable on the left.