2.5 More Problem Solving

A. Two Similar Unknowns

When you have two unknowns that both add to a number, say 100, we can write them as x and $100 - x$

Reason: For two unknowns x and y, if $x + y = 100$, then $y = 100 - x$

B. Mixture Problems

Whenever we have two types of items that we put together to form a third, we have a **mixture problem**. The general equation for a mixture problem is:

"stuff1+stuff2 =total stuff"

Often in mixture problems we will need to make use of the "two similar unknowns" principle.

C. Examples of Mixture Problems

Example 1: How much 70% solution and 40% solution need to be mixed to get 50 gallons of 50% solution?

Solution

Use the "stuff1+stuff2 =total stuff" equation:

70% solution of unknown1 amount $+40\%$ solution of unknown2 amount $= 50\%$ solution of 50 gallons

2. Let x =gallons of 70% solution

By the two similar unknowns principle, $50 - x$ =gallons of 40% solution.

70% of $x + 40\%$ of $(50 - x) = 50\%$ of 50

3. $.70x + .40(50 - x) = .50(50)$

4. $.70x + 20 - .40x = 25 \;\Rightarrow\; .30x = 5 \;\Rightarrow\; x = \frac{5}{.30} \approx 16.7$

5. Answer the question!

   We've found the gallons of 70% solution. Now $50 - x \approx 50 - 16.7 = 33.3$ is the number of gallons of 40% solution

Ans $\boxed{16.7 \text{ gallons of } 70\% \text{ solution and } 33.3 \text{ gallons of } 40\% \text{ solution}}$

Example 2: How much candy costing \$ 5.29 per pound should be mixed with 5 pounds of candy costing \$ 3.69 per pound to yield a mixture worth \$ 4.60 per pound?

Solution

Use the "stuff1+stuff2 =total stuff" equation:

\$ 5.29 per pound of unknown amount $+$ \$ 3.69 per pound of 5 pounds $=$ \$ 4.60 per pound of total mixture

2. Let x =amount of \$ 5.29 per pound candy

Adding 5 pounds to x yields a total mixture of $x + 5$ pounds.

5.29 of $x + 3.69$ of $5 = 4.60$ of $(x + 5)$

3. $5.29x + 3.69(5) = 4.60(x + 5)$

4. $5.29x + 18.45 = 4.60x + 23 \;\Rightarrow\; .69x = 4.55 \;\Rightarrow\; x = \frac{4.55}{.69} \approx 6.59$

Ans $\boxed{\text{Approximately } 6.59 \text{ pounds of \$ } 5.29 \text{ per pound candy should be added}}$

D. Distance/Rate/Time Problems

Any problem involving motion is called a **distance/rate/time** problem. The fundamental relationship between distance/rate/time is $d = rt$.

The **usual** strategy for solving these problems is to ask the question: What do you know about distance at the end of the problem?

E. An Example

Two cross-country skiers start skiing at the same time on the same trail going in the same direction. The more experienced skier averages 6 miles per hour, while the beginning skier averages 2 miles per hour. After how many hours of skiing will the two skiers be 10 miles apart?

Solution

1. Experienced skier= 6mph

   Beginning skier= 2mph

   End of the problem: $d_{experienced} - d_{beginning} = 10$

2. Let t =number of hours skiing

   Then using $d = rt$, $d_{experienced} = 6t$ and $d_{beginning} = 2t$

3. $6t - 2t = 10$

4. $4t = 10 \;\Rightarrow\; t = \frac{10}{4} = 2.5$

Ans $\boxed{2.5 \text{ hours}}$