2.3 Absolute Value Equations

A. Absolute Value

Recall that $|\cdot|$ means **distance from the origin**.

B. Strategy

1. Draw a number line and mark the location(s) that have the required distance.

2. Rewrite the problem without absolute value signs using the marked locations and solve.

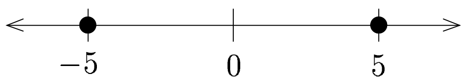
Note: Most of the time, the problem breaks into **two** equations.

C. Examples

Example 1: Solve $|2x+3| = 5$ for x

Solution

1. Distance from the origin$= 5$



2. Now write the new equations:

$2x + 3 = -5$ OR $2x + 3 = 5$

$2x + 3 \underline{-3} = -5 \underline{-3}$ OR $2x + 3 \underline{-3} = 5 \underline{-3}$

$2x = -8 \quad \text{OR} \quad 2x = 2$

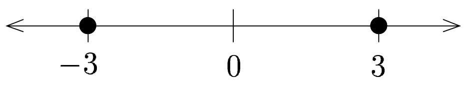
$\frac{2x}{2} = \frac{-8}{2} \quad \text{OR} \quad \frac{2x}{2} = \frac{2}{2}$

Ans $\boxed{x = -4 \quad \text{OR} \quad x = 1}$

Example 2: Solve $|7 - x| = 3$ for x

Solution

1. Distance from the origin$= 3$



2. Now write the new equations:

$7 - x = -3 \quad \text{OR} \quad 7 - x = 3$

$7 - x \; \underline{-7} = -3 \; \underline{-7} \quad \text{OR} \quad 7 - x \; \underline{-7} = 3 \; \underline{-7}$

$-x = -10 \quad \text{OR} \quad -x = -4$

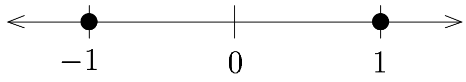
$\frac{-x}{-1} = \frac{-10}{-1} \quad \text{OR} \quad \frac{-x}{-1} = \frac{-4}{-1}$

Ans $\boxed{x = 10 \quad \text{OR} \quad x = 4}$

Example 3: Solve $\left|\frac{5x}{3} - \frac{7}{2}\right| = 1$ for x

Solution

1. Distance from the origin$= 1$



2. Now write the new equations:

$\frac{5x}{3} - \frac{7}{2} = -1$ OR $\frac{5x}{3} - \frac{7}{2} = 1$

$6\left(\frac{5x}{3} - \frac{7}{2}\right) = 6(-1)$ OR $6\left(\frac{5x}{3} - \frac{7}{2}\right) = 6(1)$

$10x - 21 = -6$ OR $10x - 21 = 6$

$10x - 21 \underline{+21} = -6 \underline{+21}$ OR $10x - 21 \underline{+21} = 6 \underline{+21}$

$10x = 15$ OR $10x = 27$

$\frac{10x}{10} = \frac{15}{10}$ OR $\frac{10x}{10} = \frac{27}{10}$

Ans $\boxed{x = \frac{3}{2} \quad \text{OR} \quad x = \frac{27}{10}}$

D. Comments

1. To check your solution, plug **each** number back in to the equation. **Both** numbers must work for a correct solution.

2. Always draw the number line and use "distance"; don't just use some "$+/-$" rule, which **fails** if the right hand side is negative.

Think about how to do these:

1. Solve $|2x - 3| = -4$ for x

2. Solve $|2x - 3| = 0$ for x