2.1C Special Cases

A. Discussion

Sometimes when solving an equation, "x" **disappears**!

If you get something **false**, like $0 = 3$, $-1 = 4$, etc., then the equation has **no solutions**. This is sometimes written as $\emptyset$.

If you get something **true**, like $0 = 0$, $5 = 5$, etc., then the equation is **always true**.
Every real number works. The solution is "all real numbers".
This is sometimes written as $(-\infty, \infty)$.

B. Examples

Example 1: Solve $2x - 4 = 2(3 + x)$ for x

Solution

1. Simplify:

   Clear parentheses: $2x - 4 = 6 - 2x$

   No fractions to clear

   No like terms to combine

2. Isolate x:

$$2x + 4 \underline{-2x} = 6 + 2x \underline{-2x}$$

$$-4 = 6$$

This is a **false** statement. Thus the equation has **no solutions**.

Ans $\boxed{\emptyset}$

Example 2: Solve $5x - 3(x - 2) = 2(x + 3)$ for x

Solution

1. Simplify:

   Clear parentheses: $5x - 3x + 6 = 2x + 6$

   Collect like terms: $2x + 6 = 2x + 6$

2. Isolate x:

$$2x + 6 \underline{-2x} = 2x + 6 \underline{-2x}$$

$$6 = 6$$

This is a **true** statement. Thus the solution is **all real numbers**.

Ans $\boxed{(-\infty, \infty)}$

Note: An equation with “all real numbers” as a solution is an identity. You can check an identity by plugging in **any** random number you want into the original equation. It has to be true for **all** such random picks.

For instance, in Example 2, the number 3 should work. Let’s check it:

$$5(3) - 3(3-2) \stackrel{?}{=} 2(3+3)$$

$$15 - 3(1) \stackrel{?}{=} 2 \cdot 6$$

$$15 - 3 \stackrel{?}{=} 12$$

It checks!