

1.4B Properties of Exponents II

A. Summary of Rules

1. **Product Rule:** $x^m x^n = x^{m+n}$ multiply powers \Rightarrow **add exponents**
2. **Quotient Rule:** $\frac{x^m}{x^n} = x^{m-n}$ divide powers \Rightarrow **subtract exponents**
3. **Power Rule:** $(x^m)^n = x^{mn}$ power of a power \Rightarrow **multiply exponents**
4. **Zero Power:** $x^0 = 1$, if $x \neq 0$
5. **Negative Exponents:** $x^{-n} = \frac{1}{x^n}$ negative exponent \Rightarrow **reciprocal**
6. **Switch Rule:** $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$ “up goes down, down goes up”
7. **Multiple Power Rule:** $\left(\frac{xy}{z}\right)^m = \frac{x^m y^m}{z^m}$ “hit every entry”

B. Simplifying

We now want to be able to simplify expressions using several of these rules in one problem.

To simplify, we do the following:

1. Use product and quotient rules to compress the expression.
2. Use power rules to “clear” parentheses.
3. Get rid of negative exponents using rules.

C. Examples

Example 1: Simplify $(4x^2yz^3)^3$

Solution

$$(4x^2yz^3)^3 = 4^3 x^6 y^3 z^9 \text{ (using the power rule)}$$

Ans $\boxed{64x^6y^3z^9}$

Example 2: Simplify $(2x^{-2}y^3z^{-1})^{-5}$

Solution

$$(2x^{-2}y^3z^{-1})^{-5} = 2^{-5} x^{10} y^{-15} z^5 \text{ (using the power rule)}$$

Now get rid of the negative exponents:

$$\frac{x^{10} z^5}{2^5 y^{15}}$$

Ans $\boxed{\frac{x^{10} z^5}{32y^{15}}}$

Example 3: Simplify $\left(\frac{6x^2y^{-1}}{3x^{-5}y^2z^{-1}}\right)^{-2}$

Solution

Simplify the inside first to avoid big numbers:

$$\left(\frac{6x^2y^{-1}}{3x^{-5}y^2z^{-1}}\right)^{-2} = \left(\frac{2x^7y^{-3}}{z^{-1}}\right)^{-2}$$

Now clear parentheses:

$$\frac{2^{-2}x^{-14}y^6}{z^2}$$

Switch rule to get rid of negative exponents:

$$\frac{y^6}{2^2x^{14}z^2}$$

Ans $\boxed{\frac{y^6}{4x^{14}z^2}}$

Example 4: Simplify $\left(\frac{14x^{-2}y^{-3}z^{-4}}{21x^{-6}y^2z^{-4}w^2}\right)^{-2}$

Solution

Simplify the inside: $\frac{14}{21} = \frac{2}{3}$ and use quotient rule

$$\left(\frac{2x^4y^{-5}z^0}{3w^2}\right)^{-2}$$

Now clear parentheses using the power rule:

$$\frac{2^{-2}x^{-8}y^{10}z^0}{3^{-2}w^{-4}}$$

Now get rid of negative exponents using the switch rule:

$$\frac{3^2y^{10}z^0w^4}{2^2x^8}$$

Simplify powers to get the answer

Ans $\boxed{\frac{9y^{10}w^4}{4x^8}}$