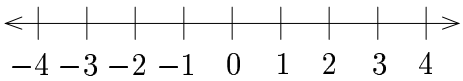
1.2B Real Numbers/Signed Numbers

A. Number Line



$<$	less than	$\leq$	less than or equal to
$>$	greater than	$\geq$	greater than or equal to

Examples:

$-1 < 2$

$-3 \leq -1$

$0 > -2$

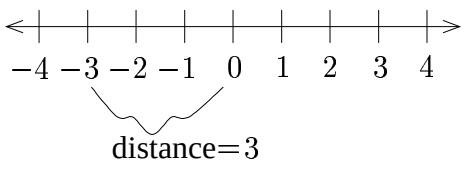
$-1 \geq -1$

B. Absolute Value

$|\cdot|$ means **distance from the origin** (origin is 0)

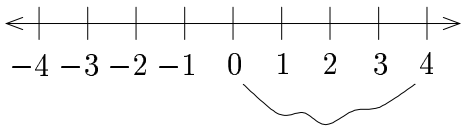
Examples:

1. Find $|-3|$



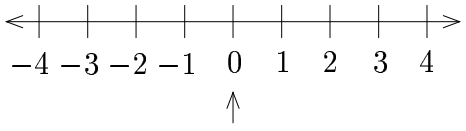
$|-3| = \boxed{3}$

2. Find $|4|$



$|4| = \boxed{4}$

3. Find $|0|$



$|0| = \boxed{0}$

It "looks" like absolute value just means "make it positive". While this is true for numbers, we need the distance concept for more difficult problems.

C. Adding Signed Numbers

If the signs are the **same**, we "add" and keep the sign.

If the signs are **different**, we "subtract" and keep the sign of the number with larger magnitude (i.e. larger absolute value)

Hint: Check to see if your answers make sense. Think of positive numbers as "money" and negative numbers as "debt".

Examples:

1. Find $4 + 2$

   same sign (both positive), so add: $\boxed{6}$

2. Find $-3 + -4$

   same sign (both negative), so add: $\boxed{-7}$

3. Find $6 + (-2)$

   different signs (one pos., one neg.), so subtract $(6 - 2)$ and keep $+$ sign: $\boxed{4}$

4. Find $6 + (-14)$

   different signs, so subtract $(14 - 6)$ and keep $-$ sign: $\boxed{-8}$

5. Find $-3 + 5$

   different signs, so subtract $(5 - 3)$ and keep $+$ sign: $\boxed{2}$

D. Subtracting Signed Numbers

To subtract, switch the sign of the second number and then add.

Examples:

1. Find $-3 - 4$

   $-3 - 4 = -3 + (-4)$ Now add (same sign rule): $\boxed{-7}$

2. Find $2 - (-3)$

   $2 - (-3) = 2 + 3$ Now add (same sign rule): $\boxed{5}$

3. Find $4 - 8$

   $4 - 8 = 4 + (-8)$ Now add (different sign rule: $8 - 4$ with $-$): $\boxed{-4}$

4. Find $-3-(-4)$

$-3-(-4)=-3+4$ Now add (different sign rule: $4-3$ with $+$): $\boxed{1}$

E. Multiplying/Dividing Signed Numbers

If the two numbers have the **same sign**, the answer is **positive**.

If the two numbers have **different signs**, the answer is **negative**.

Examples:

1. Find $(-3)(5)$

   different signs, so answer is negative: $\boxed{-15}$

2. Find $-14 \div -7$

   same signs, so the answer is positive: $\boxed{2}$

3. Find $\frac{20}{-5}$

   different signs, so answer is negative: $\boxed{-4}$

4. Find $(-5)(-6)$

   same signs, so the answer is positive: $\boxed{30}$

WARNING: Pay attention to parentheses

$-3-8$ is subtraction, but $(-3)(-8)$ is multiplication

F. Division with Zero

Zero divided by anything nonzero is 0.

Thus $0 \div 5$, $0 \div -3$, $\frac{0}{4}$ are all 0.

Anything divided by zero is **undefined**.

Thus $5 \div 0$, $-3 \div 0$, $\frac{4}{0}$ are all undefined.

THOUGHT QUESTION: Why can't we divide by zero?

An easy rule to remember regarding zero is this:

Zero on top is zero, zero on bottom is undefined

Note: The special case $\frac{0}{0}$ is called **indeterminate**.

G. Signs and Fractions

One minus sign in a fraction can be moved to the top, bottom, or front: YOUR CHOICE.

Thus all of these are the same: $\frac{-2}{3}$, $\frac{2}{-3}$, $-\frac{2}{3}$

Two minus signs in a fraction is the same as no minus signs.

Thus these are the same: $\frac{-2}{-3}$ and $\frac{2}{3}$

H. Arithmetic with Signed Fractions

With addition and subtraction, first convert each fraction to its LCD. Then when you add/subtract numerators, you use the signed number rules.

With multiplication and division, multiply or divide as usual, but remember the signed number rules.

Examples:

1. Find $\frac{5}{2} \div -\frac{3}{2}$

$$\frac{5}{2} \div \frac{-3}{2} = \frac{5}{2} \cdot \frac{2}{-3} = \frac{5}{\cancel{2}_{1}} \cdot \frac{\overset{1}{\cancel{2}}}{-3} = \frac{5}{-3} = \boxed{-\frac{5}{3}}$$

2. Find $-\frac{3}{8} + \frac{1}{6}$

   Find $\mathcal{lcm}(8, 6)$: 8, 16, 24 (stop)

   Then $\frac{-3}{8} + \frac{1}{6} = \frac{-9}{24} + \frac{4}{24}$

   Now use add (using the same sign rule): $\frac{-5}{24} = \boxed{-\frac{5}{24}}$

3. Find $\dfrac{-\frac{4}{5}}{-\frac{6}{25}}$

$$\frac{-\frac{4}{5}}{-\frac{6}{25}} = \frac{-4}{5} \div \frac{-6}{25} = \frac{-4}{5} \cdot \frac{25}{-6} = \frac{\overset{-2}{\cancel{-4}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{5}{\cancel{25}}}{\underset{-3}{\cancel{-6}}} = \frac{-10}{-3} = \boxed{\frac{10}{3}}$$

I. Order of Operations I

In problems with many operations, it is important to know what to do first.

Do all multiplications and divisions left to right, then do the additions and subtractions left to right.

To do this, just do one operation, and recopy the problem. Then repeat until you get an answer.

Comments:

1. Multiplication and Division are "equal". If division is seen first (left to right) then do that before multiplication. The same comment is true for addition and subtraction.

2. If you see a fraction bar, treat the top and bottom separately. Then simplify the fraction at the very end.

Examples:

1. Find $2 - 6 \div 2 \cdot 5$

Do all multiplications and divisions left to right, doing one step and recopying each time:

$$2 - 6 \div 2 \cdot 5$$

$$2 - 3 \cdot 5$$

$$2 - 15$$

Now do the remaining subtraction:

$$2 - 15 = 2 + (-15) = \boxed{-13}$$

2. Find $3 + (4)(-2) - 18 \div 3 \cdot 2$

Do all multiplications and divisions left to right, doing one step and recopying each time:

$$3 + (4)(-2) - 18 \div 3 \cdot 2$$

$$3 + (-8) - 18 \div 3 \cdot 2$$

$$3 + (-8) - 6 \cdot 2$$

$$3 + (-8) - 12$$

Now do all the additions and subtractions left to right:

$$-5 - 12 = -5 + (-12) = \boxed{-17}$$

3. Find $\dfrac{-8 + (-3)(-4) + 20 \div 2 \cdot 5}{|(-4)(-3) - 4 - 3 \cdot 4|}$

Keeping the top and bottom separate, do all multiplications and divisions left to right:

$$\frac{-8 + (-3)(-4) + 20 \div 2 \cdot 5}{|(-4)(-3) - 4 - 3 \cdot 4|}$$

$$\frac{-8 + 12 + 20 \div 2 \cdot 5}{|12 - 4 - 3 \cdot 4|}$$

$$\frac{-8 + 12 + 10 \cdot 5}{|12 - 4 - 12|}$$

$$\frac{-8 + 12 + 50}{|12 - 4 - 12|}$$

Now do all the additions and subtractions left to right:

$$\frac{4+50}{|8-12|}$$

$$\frac{4+50}{|8+(-12)|}$$

$$\frac{54}{|-4|}$$

$\frac{54}{4} = \boxed{\frac{27}{2}}$