1.2A Fractions Review

A. Simplifying

To simplify a fraction, we divide top and bottom by common factors. If we choose the largest (greatest common factor), then we can do it in one step.

Examples:

1. Simplify $\frac{10}{18}$

$$\frac{10}{18} \xrightarrow[\div 2]{\div 2} \boxed{\frac{5}{9}}$$

2. Simplify $\frac{16}{24}$

$$\frac{16}{24} \xrightarrow[\div 4]{\div 4} \frac{4}{6} \xrightarrow[\div 2]{\div 2} \boxed{\frac{2}{3}}$$

If you divide by the GCF$= 8$, you can do it one step: $\frac{16}{24} \xrightarrow[\div 8]{\div 8} \boxed{\frac{2}{3}}$

B. Multiplying and Dividing I

1. To multiply: multiply numerators and multiply denominators; parentheses mean multiply

2. To divide: invert second (reciprocal) and multiply; fraction bar means divide

Examples:

1. Find $\left(\frac{2}{13}\right)\left(\frac{3}{5}\right)$

$$\left(\frac{2}{13}\right)\left(\frac{3}{5}\right) = \frac{2}{13} \cdot \frac{3}{5} = \boxed{\frac{6}{65}}$$

2. Find $\frac{6}{5} \div \frac{7}{3}$

$$\frac{6}{5} \div \frac{7}{3} = \frac{6}{5} \cdot \frac{3}{7} = \boxed{\frac{18}{35}}$$

3. Find $\frac{\frac{7}{2}}{\frac{3}{5}}$

$$\frac{\frac{7}{2}}{\frac{3}{5}} = \frac{7}{2} \div \frac{3}{5} = \frac{7}{2} \cdot \frac{5}{3} = \boxed{\frac{35}{6}}$$

Note: In algebra, we leave fractions **improper**. We don't convert to mixed numbers.

C. Multiplying and Dividing II

Sometimes we need to simplify after multiplying. A shortcut is to **cancel** first. We can cancel common factors between any numerator and any denominator when *multiplying*.

Note: Never cancel horizontally; only cancel vertically or diagonally.

Examples:

1. Find $\frac{2}{3} \cdot \frac{9}{10}$

$$\frac{2}{3} \cdot \frac{9}{10} = \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \cdot \frac{\cancel{9}^{3}}{\cancel{10}_{5}} = \boxed{\frac{3}{5}}$$

2. Find $\frac{10}{18} \div \frac{5}{9}$

$$\frac{10}{18} \div \frac{5}{9} = \frac{10}{18} \cdot \frac{9}{5} = \frac{\cancel{10}^{1}}{\cancel{18}_{1}} \cdot \frac{\cancel{9}^{1}}{\cancel{5}_{1}} = \frac{1}{1} = \boxed{1}$$

D. Adding and Subtracting I

With the **same** denominators, we add/subtract numerators. Then simplify.

Examples:

1. Find $\frac{5}{3}+\frac{2}{3}$

   $\frac{5}{3}+\frac{2}{3}=\boxed{\frac{7}{3}}$

2. Find $\frac{5}{6}-\frac{1}{6}$

   $\frac{5}{6}-\frac{1}{6}=\frac{4}{6}\xrightarrow[\div 2]{\div 2}\boxed{\frac{2}{3}}$

3. Find $\frac{9}{2}+\frac{3}{2}$

   $\frac{9}{2}+\frac{3}{2}=\frac{12}{2}\xrightarrow[\div 2]{\div 2}\frac{6}{1}=\boxed{6}$

E. Least Common Multiple

Goal: Find the smallest number that is a multiple of two numbers

Method 1: (the second method will come later in the course)

Take the larger number and keep adding it to itself until it is a multiple of the smaller number.

Examples:

1. Find $\mathscr{lcm}(6,8)$

   8, 16, 24 (STOP: 24 is a multiple of 6) $\boxed{24}$

2. Find $\ell cm(10, 8)$

10, 20, 30, 40 (STOP: 40 is a multiple of 8) $\boxed{40}$

F. Adding and Subtracting II

With **different** denominators:

1. Find the LCM of the denominators (called the **least common denominator** or **LCD**)

2. Rewrite each fraction with the LCD as the new denominator. Do this by multiplying top and bottom of the original fraction by the "needed" number.

3. Now add/subtract as in Part D.

Examples:

1. Find $\frac{1}{6} + \frac{3}{8}$

First find $\ell cm(6, 8)$:

8, 16, 24 (STOP: 24 is a multiple of 6), so $\ell cm(6, 8) = 24$

Now rewrite each fraction with 24 as the new denominator:

$$\frac{1}{6} + \frac{3}{8} \rightsquigarrow \frac{}{\frac{24}{6\cdot 4}} + \frac{}{\frac{24}{8\cdot 3}} \rightsquigarrow \frac{1\cdot 4}{\frac{24}{6\cdot 4}} + \frac{3\cdot 3}{\frac{24}{8\cdot 3}} = \frac{4}{24} + \frac{9}{24} = \boxed{\frac{13}{24}}$$

Note: For the first fraction, we multiplied 6 by 4 to get 24, so that is what we multiply the top by. For the second fraction, we multiplied 8 by 3 to get 24, so that is what we multiply the top by.

2. Find $\frac{3}{2} - \frac{1}{3}$

First find $\ell cm(2, 3)$:

3, 6 (STOP: 6 is a multiple of 2), so $\ell cm(2, 3) = 6$

Now rewrite each fraction with 6 as the new denominator:

$$\frac{3}{2} - \frac{1}{3} \rightsquigarrow \underset{2\cdot 3}{\frac{}{6}} - \underset{3\cdot 2}{\frac{}{6}} \rightsquigarrow \underset{2\cdot 3}{\frac{3\cdot 3}{6}} - \underset{3\cdot 2}{\frac{1\cdot 2}{6}} = \frac{9}{6} - \frac{2}{6} = \boxed{\frac{7}{6}}$$

3. Find $\frac{7}{30} + \frac{1}{14}$

First find $\ell cm(30, 14)$:

30, 60, 90, 120, 150, 180, 210

(STOP: 210 is a multiple of 14), so $\ell cm(14, 30) = 210$

Now rewrite each fraction with 210 as the new denominator:

$$\frac{7}{30} + \frac{1}{14} \rightsquigarrow \underset{30\cdot 7}{\frac{}{210}} + \underset{14\cdot 15}{\frac{}{210}} \rightsquigarrow \underset{30\cdot 7}{\frac{7\cdot 7}{210}} + \underset{14\cdot 15}{\frac{1\cdot 15}{210}} = \frac{49}{210} + \frac{15}{210} = \frac{64}{210} \xrightarrow[\div 2]{\div 2} \boxed{\frac{32}{105}}$$